

In recent years, increasing applications as communications, computing, printing, and energy. *Fundamentals of* self-contained textbook to offer of this rapidly engineering and ap

Featuring a logic and applications, detailed accounts of theories of light, wave optics, electrodynamics, and photon optics, interaction of light with matter, theory of semiconductors, and their optical properties. Presented at increasing complexity, these serve as building blocks for more advanced topics in optics and photonics. Wave and fiber optics, sources and detectors, and acousto-optic devices, near optical devices, communications, and computing. In vital topics as:

- Generation of lasers, and incandescence and light-emitting diodes
- Transmission of light through optical components (lenses, mirrors, and imaging systems), waveguides, and fibers
- Modulation, switching, and routing of light through optoelectronic devices, electrically, acoustically, and optically controlled devices
- Amplification and conversion of light by nonlinear interactions in nonlinear media
- Detection of light by photodiodes, phototubes, and photomultiplier tubes

Each chapter contains highlighted equations and exercises, and lists. Examples of ray tracing are included to emphasize the governing applications of interest, and appendices give the properties of two-dimensional Fourier transforms, linear systems theory, and linear systems.

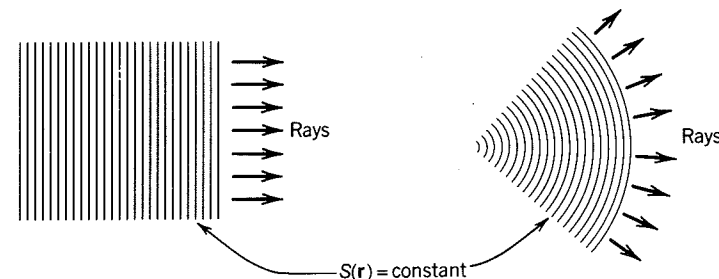


Figure 1.3-10 Rays and surfaces of constant  $S(r)$  in a homogeneous medium.

which is usually written in the vector form

$$|\nabla S|^2 = n^2,$$

(1.3-18)  
Eikonal Equation

where  $|\nabla S|^2 = \nabla S \cdot \nabla S$ . The proof of the eikonal equation from Fermat's principle is a mathematical exercise that lies beyond the scope of this book.<sup>†</sup> Fermat's principle (and the ray equation) can also be shown to follow from the eikonal equation. Therefore, either the eikonal equation or Fermat's principle may be regarded as the principal postulate of ray optics.

Integrating the eikonal equation (1.3-18) along a ray trajectory between two points  $A$  and  $B$  gives

$$S(\mathbf{r}_B) - S(\mathbf{r}_A) = \int_A^B |\nabla S| ds = \int_A^B n ds = \text{optical path length between } A \text{ and } B.$$

This means that the difference  $S(\mathbf{r}_B) - S(\mathbf{r}_A)$  represents the optical path length between  $A$  and  $B$ . In the electrostatics analogy, the optical path length plays the role of the potential difference.

To determine the ray trajectories in an inhomogeneous medium of refractive index  $n(\mathbf{r})$ , we can either solve the ray equation (1.3-2), as we have done earlier, or solve the eikonal equation for  $S(\mathbf{r})$ , from which we calculate the gradient  $\nabla S$ .

If the medium is homogeneous, i.e.,  $n(\mathbf{r})$  is constant, the magnitude of  $\nabla S$  is constant, so that the wavefront normals (rays) must be straight lines. The surfaces  $S(\mathbf{r}) = \text{constant}$  may be parallel planes or concentric spheres, as illustrated in Fig. 1.3-10.

## 1.4 MATRIX OPTICS

Matrix optics is a technique for tracing paraxial rays. The rays are assumed to travel only within a single plane, so that the formalism is applicable to systems with planar geometry and to meridional rays in circularly symmetric systems.

A ray is described by its position and its angle with respect to the optical axis. These variables are altered as the ray travels through the system. In the paraxial approximation, the position and angle at the input and output planes of an optical system are

<sup>†</sup>See, e.g., M. Born and E. Wolf, *Principles of Optics*, Pergamon Press, New York, 6th ed. 1980.

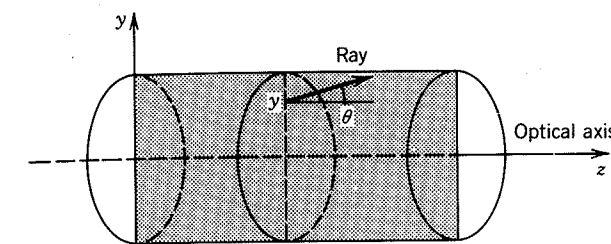


Figure 1.4-1 A ray is characterized by its coordinate  $y$  and its angle  $\theta$ .

related by two linear algebraic equations. As a result, the optical system is described by a  $2 \times 2$  matrix called the ray-transfer matrix.

The convenience of using matrix methods lies in the fact that the ray-transfer matrix of a cascade of optical components (or systems) is a product of the ray-transfer matrices of the individual components (or systems). Matrix optics therefore provides a formal mechanism for describing complex optical systems in the paraxial approximation.

### A. The Ray-Transfer Matrix

Consider a circularly symmetric optical system formed by a succession of refracting and reflecting surfaces all centered about the same axis (optical axis). The  $z$  axis lies along the optical axis and points in the general direction in which the rays travel. Consider rays in a plane containing the optical axis, say the  $y$ - $z$  plane. We proceed to trace a ray as it travels through the system, i.e., as it crosses the transverse planes at different axial distances. A ray crossing the transverse plane at  $z$  is completely characterized by the coordinate  $y$  of its crossing point and the angle  $\theta$  (Fig. 1.4-1).

An optical system is a set of optical components placed between two transverse planes at  $z_1$  and  $z_2$ , referred to as the input and output planes, respectively. The system is characterized completely by its effect on an incoming ray of arbitrary position and direction  $(y_1, \theta_1)$ . It steers the ray so that it has new position and direction  $(y_2, \theta_2)$  at the output plane (Fig. 1.4-2).

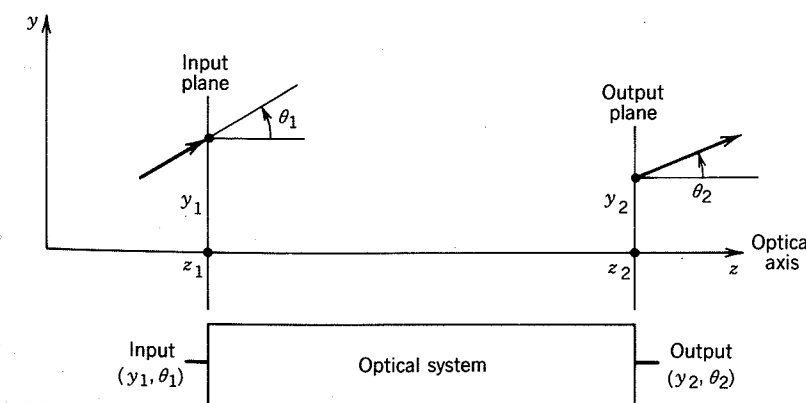


Figure 1.4-2 A ray enters an optical system at position  $y_1$  and angle  $\theta_1$  and leaves at position  $y_2$  and angle  $\theta_2$ .

In recent years, with the increasing use of computers in communications, computing, and printing, and the development of self-contained textbooks to meet the needs of this rapid engineering advancement.

Featuring a wide range of applications and detailed analyses of the theories of wave optics and photonics, the interaction of light with matter, and their representation at the complexity of the building blocks of modern optics, wave and quantum optics, and acoustics, optical communication, and computer vision topics.

- **Generation of light**—lasers, light-emitting diodes, and other light sources
- **Transmission**—optical communication, fiber optics, and integrated optics
- **Modulation**—modulation of light by electrical and optical means
- **Amplification**—optical amplifiers and lasers
- **Detection**—optical detectors and sensors

Each chapter highlights the key concepts and exercises. Examples are included to illustrate the governing principles, and the problems are of varying difficulty to test the student's understanding of the linear system analysis.

In the paraxial approximation, when all angles are sufficiently small so that  $\sin \theta \approx \theta$ , the relation between  $(y_2, \theta_2)$  and  $(y_1, \theta_1)$  is linear and can generally be written in the form

$$y_2 = Ay_1 + B\theta_1 \quad (1.4-1)$$

$$\theta_2 = Cy_1 + D\theta_1, \quad (1.4-2)$$

where  $A$ ,  $B$ ,  $C$  and  $D$  are real numbers. Equations (1.4-1) and (1.4-2) may be conveniently written in matrix form as

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}.$$

The matrix  $\mathbf{M}$ , whose elements are  $A, B, C, D$ , characterizes the optical system completely since it permits  $(y_2, \theta_2)$  to be determined for any  $(y_1, \theta_1)$ . It is known as the **ray-transfer matrix**.

#### EXERCISE 1.4-1

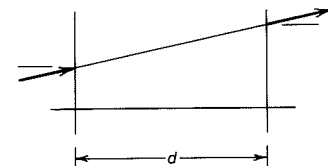
**Special Forms of the Ray-Transfer Matrix.** Consider the following situations in which one of the four elements of the ray-transfer matrix vanishes:

- Show that if  $A = 0$ , all rays that enter the system at the same angle leave at the same position, so that parallel rays in the input are focused to a single point at the output.
- What are the special features of each of the systems for which  $B = 0$ ,  $C = 0$ , or  $D = 0$ ?

### B. Matrices of Simple Optical Components

#### Free-Space Propagation

Since rays travel in free space along straight lines, a ray traversing a distance  $d$  is altered in accordance with  $y_2 = y_1 + \theta_1 d$  and  $\theta_2 = \theta_1$ . The ray-transfer matrix is therefore

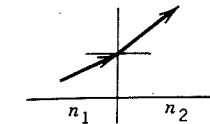


$$\mathbf{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}. \quad (1.4-3)$$

#### Refraction at a Planar Boundary

At a planar boundary between two media of refractive indices  $n_1$  and  $n_2$ , the ray angle changes in accordance with Snell's law  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ . In the paraxial approximation,  $n_1 \theta_1 \approx n_2 \theta_2$ . The position of the ray is not altered,  $y_2 = y_1$ . The ray-transfer

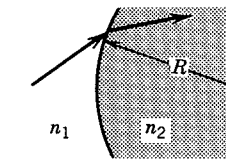
matrix is



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}. \quad (1.4-4)$$

#### Refraction at a Spherical Boundary

The relation between  $\theta_1$  and  $\theta_2$  for paraxial rays refracted at a spherical boundary between two media is provided in (1.2-8). The ray height is not altered,  $y_2 \approx y_1$ . The ray-transfer matrix is

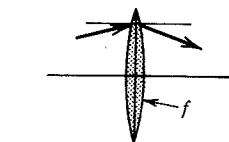


$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}. \quad (1.4-5)$$

Convex,  $R > 0$ ; concave,  $R < 0$

#### Transmission Through a Thin Lens

The relation between  $\theta_1$  and  $\theta_2$  for paraxial rays transmitted through a thin lens of focal length  $f$  is given in (1.2-11). Since the height remains unchanged ( $y_2 = y_1$ ),



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}. \quad (1.4-6)$$

Convex,  $f > 0$ ; concave,  $f < 0$

#### Reflection from a Planar Mirror

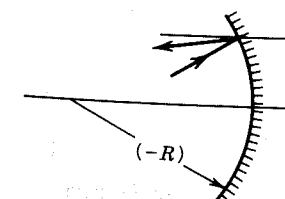
Upon reflection from a planar mirror, the ray position is not altered,  $y_2 = y_1$ . Adopting the convention that the  $z$  axis points in the general direction of travel of the rays, i.e., toward the mirror for the incident rays and away from it for the reflected rays, we conclude that  $\theta_2 = \theta_1$ . The ray-transfer matrix is therefore the identity matrix



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (1.4-7)$$

#### Reflection from a Spherical Mirror

Using (1.2-1), and the convention that the  $z$  axis follows the general direction of the rays as they reflect from mirrors, we similarly obtain



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}. \quad (1.4-8)$$

Concave,  $R < 0$ ; convex,  $R > 0$

In recent years, there has been an increasing application of optics in communications, computing, printing, and other fields. This book is a self-contained textbook to one of this rapidly growing engineering and science.

Featuring a lot of applications and detailed accounts of theories of light wave optics, electron optics, and photon optics, interaction of light with matter, and their optical properties, these building blocks lead to more advanced topics in fiber optics and wave and fiber optics, and acousto-optics, and optical devices, communications, and computing vital topics as:

- Generation of lasers, and incoherent light-emitting diodes.
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- Detection of light by semiconductor photodiodes.

Each chapter contains highlighted equations and exercises, and lists. Examples are included to emphasize governing applications, and the properties of linear systems of linear systems.

Note the similarity between the ray-transfer matrices of a spherical mirror (1.4-8) and a thin lens (1.4-6). A mirror with radius of curvature  $R$  bends rays in a manner that is identical to that of a thin lens with focal length  $f = -R/2$ .

### C. Matrices of Cascaded Optical Components

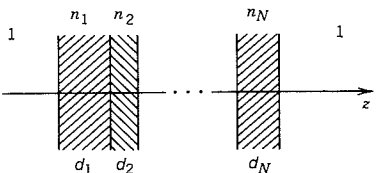
A cascade of optical components whose ray-transfer matrices are  $M_1, M_2, \dots, M_N$  is equivalent to a single optical component of ray-transfer matrix

$$\begin{array}{c} \rightarrow \\ \boxed{M_1} \rightarrow \boxed{M_2} \rightarrow \dots \rightarrow \boxed{M_N} \rightarrow \end{array} \quad M = M_N \cdots M_2 M_1. \quad (1.4-9)$$

Note the order of matrix multiplication: The matrix of the system that is crossed by the rays first is placed to the right, so that it operates on the column matrix of the incident ray first.

#### EXERCISE 1.4-2

**A Set of Parallel Transparent Plates.** Consider a set of  $N$  parallel planar transparent plates of refractive indices  $n_1, n_2, \dots, n_N$  and thicknesses  $d_1, d_2, \dots, d_N$ , placed in air ( $n = 1$ ) normal to the  $z$  axis. Show that the ray-transfer matrix is

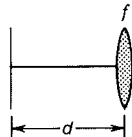


$$M = \begin{bmatrix} 1 & \sum_{i=1}^N \frac{d_i}{n_i} \\ 0 & 1 \end{bmatrix}. \quad (1.4-10)$$

Note that the order of placing the plates does not affect the overall ray-transfer matrix. What is the ray-transfer matrix of an inhomogeneous transparent plate of thickness  $d_0$  and refractive index  $n(z)$ ?

#### EXERCISE 1.4-3

**A Gap Followed by a Thin Lens.** Show that the ray-transfer matrix of a distance  $d$  of free space followed by a lens of focal length  $f$  is



$$M = \begin{bmatrix} 1 & d \\ -\frac{1}{f} & 1 - \frac{d}{f} \end{bmatrix}. \quad (1.4-11)$$

#### EXERCISE 1.4-4

**Imaging with a Thin Lens.** Derive an expression for the ray-transfer matrix of a system comprised of free space/thin lens/free space, as shown in Fig. 1.4-3. Show that if the

imaging condition ( $1/d_1 + 1/d_2 = 1/f$ ) is satisfied, all rays originating from a single point in the input plane reach the output plane at the single point  $y_2$ , regardless of their angles. Also show that if  $d_2 = f$ , all parallel incident rays are focused by the lens onto a single point in the output plane.

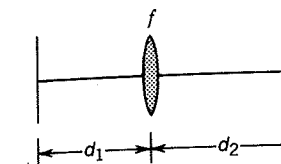


Figure 1.4-3 Single-lens imaging system.

#### EXERCISE 1.4-5

**Imaging with a Thick Lens.** Consider a glass lens of refractive index  $n$ , thickness  $d$ , and two spherical surfaces of equal radii  $R$  (Fig. 1.4-4). Determine the ray-transfer matrix of the system between the two planes at distances  $d_1$  and  $d_2$  from the vertices of the lens. The lens is placed in air (refractive index = 1). Show that the system is an imaging system (i.e., the input and output planes are conjugate) if

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f} \quad \text{or} \quad s_1 s_2 = f^2, \quad (1.4-12)$$

where

$$z_1 = d_1 + h, \quad s_1 = z_1 - f \quad (1.4-13)$$

$$z_2 = d_2 + h, \quad s_2 = z_2 - f \quad (1.4-14)$$

and

$$h = \frac{(n-1)fd}{nR} \quad (1.4-15)$$

$$\frac{1}{f} = \frac{(n-1)}{R} \left[ 2 - \frac{n-1}{n} \frac{d}{R} \right]. \quad (1.4-16)$$

The points  $F_1$  and  $F_2$  are known as the front and back focal points, respectively. The points  $P_1$  and  $P_2$  are known as the first and second principal points, respectively. Show the importance of these points by tracing the trajectories of rays that are incident parallel to the optical axis.

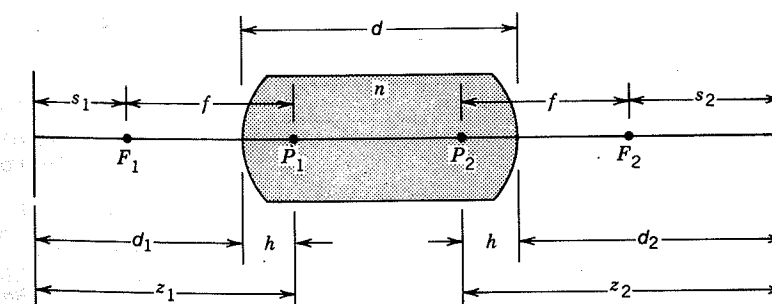


Figure 1.4-4 Imaging with a thick lens.  $P_1$  and  $P_2$  are the principal points and  $F_1$  and  $F_2$  are the focal points.

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Each chapter contains highlighted equations and exercises, lists. Examples included to emphasize governing applications, and applications the properties of linear systems linear systems.

### D. Periodic Optical Systems

A periodic optical system is a cascade of identical unit systems. An example is a sequence of equally spaced identical relay lenses used to guide light, as shown in Fig. 1.2-16(a). Another example is the reflection of light between two parallel mirrors forming an optical resonator (see Chap. 9); in that case, the ray traverses the same unit system (a round trip of reflections) repeatedly. A homogeneous medium, such as a glass fiber, may be considered as a periodic system if it is divided into contiguous identical segments of equal length. A general theory of ray propagation in periodic optical systems will now be formulated using matrix methods.

#### Difference Equation for the Ray Position

A periodic system is composed of a cascade of identical unit systems (stages), each with a ray-transfer matrix  $(A, B, C, D)$ , as shown in Fig. 1.4-5. A ray enters the system with initial position  $y_0$  and slope  $\theta_0$ . To determine the position and slope  $(y_m, \theta_m)$  of the ray at the exit of the  $m$ th stage, we apply the  $ABCD$  matrix  $m$  times,

$$\begin{bmatrix} y_m \\ \theta_m \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^m \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix}.$$

We can also apply the relations

$$y_{m+1} = Ay_m + B\theta_m \quad (1.4-17)$$

$$\theta_{m+1} = Cy_m + D\theta_m \quad (1.4-18)$$

iteratively to determine  $(y_1, \theta_1)$  from  $(y_0, \theta_0)$ , then  $(y_2, \theta_2)$  from  $(y_1, \theta_1)$ , and so on, using a computer.

It is of interest to derive equations that govern the dynamics of the position  $y_m$ ,  $m = 0, 1, \dots$ , irrespective of the angle  $\theta_m$ . This is achieved by eliminating  $\theta_m$  from (1.4-17) and (1.4-18). From (1.4-17)

$$\theta_m = \frac{y_{m+1} - Ay_m}{B}. \quad (1.4-19)$$

Replacing  $m$  with  $m + 1$  in (1.4-19) yields

$$\theta_{m+1} = \frac{y_{m+2} - Ay_{m+1}}{B}. \quad (1.4-20)$$

Substituting (1.4-19) and (1.4-20) into (1.4-18) gives

$$y_{m+2} = 2By_{m+1} - F^2y_m, \quad (1.4-21)$$

Recurrence Relation for Ray Position

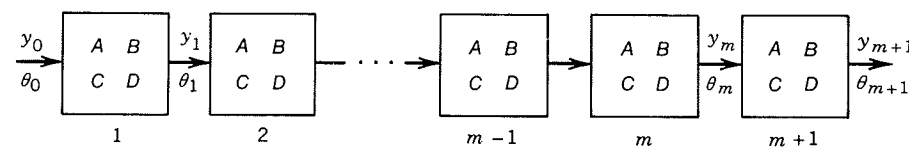


Figure 1.4-5 A cascade of identical optical components.

where

$$b = \frac{A + D}{2} \quad (1.4-22)$$

$$F^2 = AD - BC = \det[\mathbf{M}], \quad (1.4-23)$$

and  $\det[\mathbf{M}]$  is the determinant of  $\mathbf{M}$ .

Equation (1.4-21) is a linear difference equation governing the ray position  $y_m$ . It can be solved iteratively on a computer by computing  $y_2$  from  $y_0$  and  $y_1$ , then  $y_3$  from  $y_1$  and  $y_2$ , and so on.  $y_1$  may be computed from  $y_0$  and  $\theta_0$  by use of (1.4-17) with  $m = 0$ .

It is useful, however, to derive an explicit expression for  $y_m$  by solving the difference equation (1.4-21). As in linear differential equations, a solution satisfying a linear difference equation and the initial conditions is a unique solution. It is therefore appropriate to make a judicious guess for the solution of (1.4-21). We use a trial solution of the geometric form

$$y_m = y_0 h^m, \quad (1.4-24)$$

where  $h$  is a constant. Substituting (1.4-24) into (1.4-21) immediately shows that the trial solution is suitable provided that  $h$  satisfies the quadratic algebraic equation

$$h^2 - 2bh + F^2 = 0, \quad (1.4-25)$$

from which

$$h = b \pm j(F^2 - b^2)^{1/2}. \quad (1.4-26)$$

The results can be presented in a more compact form by defining the variable

$$\varphi = \cos^{-1} \frac{b}{F}, \quad (1.4-27)$$

so that  $b = F \cos \varphi$ ,  $(F^2 - b^2)^{1/2} = F \sin \varphi$ , and therefore  $h = F(\cos \varphi \pm j \sin \varphi) = F \exp(\pm j\varphi)$ , whereupon (1.4-24) becomes  $y_m = y_0 F^m \exp(\pm jm\varphi)$ .

A general solution may be constructed from the two solutions with positive and negative signs by forming their linear combination. The sum of the two exponential functions can always be written as a harmonic (circular) function, so that

$$y_m = y_{\max} F^m \sin(m\varphi + \varphi_0), \quad (1.4-28)$$

where  $y_{\max}$  and  $\varphi_0$  are constants to be determined from the initial conditions  $y_0$  and  $y_1$ . In particular,  $y_{\max} = y_0 / \sin \varphi_0$ .

The parameter  $F$  is related to the determinant of the ray-transfer matrix of the unit system by  $F = \det^{1/2}[\mathbf{M}]$ . It can be shown that regardless of the unit system,  $\det[\mathbf{M}] = n_1/n_2$ , where  $n_1$  and  $n_2$  are the refractive indices of the initial and final sections of the unit system. This general result is easily verified for the ray-transfer matrices of all the optical components considered in this section. Since the determinant of a product of two matrices is the product of their determinants, it follows that the relation  $\det[\mathbf{M}] = n_1/n_2$  is applicable to any cascade of these optical components. For example, if  $\det[\mathbf{M}_1] = n_1/n_2$  and  $\det[\mathbf{M}_2] = n_2/n_3$ , then  $\det[\mathbf{M}_2\mathbf{M}_1] = (n_2/n_3)(n_1/n_2) = n_1/n_3$ .

In most applications  $n_1 = n_2$ , so that  $\det[\mathbf{M}] = 1$  and  $F = 1$ , in which case the solution for the ray position is

$$y_m = y_{\max} \sin(m\varphi + \varphi_0). \quad (1.4-29)$$

Ray Position in  
a Periodic System

We shall assume henceforth that  $F = 1$ .

#### Condition for a Harmonic Trajectory

For  $y_m$  to be a harmonic (instead of hyperbolic) function,  $\varphi = \cos^{-1} b$  must be real. This requires that

$$|b| \leq 1 \quad \text{or} \quad \frac{|A + D|}{2} \leq 1. \quad (1.4-30)$$

Condition for a  
Stable Solution

If, instead,  $|b| > 1$ ,  $\varphi$  is then imaginary and the solution is a hyperbolic function (cosh or sinh), which increases without bound, as illustrated in Fig. 1.4-6(a). A harmonic solution ensures that  $y_m$  is bounded for all  $m$ , with a maximum value of  $y_{\max}$ . The bound  $|b| \leq 1$  therefore provides a condition of **stability** (boundedness) of the ray trajectory.

The ray angle corresponding to (1.4-29) is also a harmonic function  $\theta_m = \theta_{\max} \sin(m\varphi + \varphi_1)$ , where  $\theta_{\max}$  and  $\varphi_1$  are constants. This can be shown by use of (1.4-19) and trigonometric identities. The maximum angle  $\theta_{\max}$  must be sufficiently small so that the paraxial approximation, which underlies this analysis, is applicable.

#### Condition for a Periodic Trajectory

The harmonic function (1.4-29) is periodic in  $m$  if it is possible to find an integer  $s$  such that  $y_{m+s} = y_m$  for all  $m$ . The smallest such integer is the period. The ray then

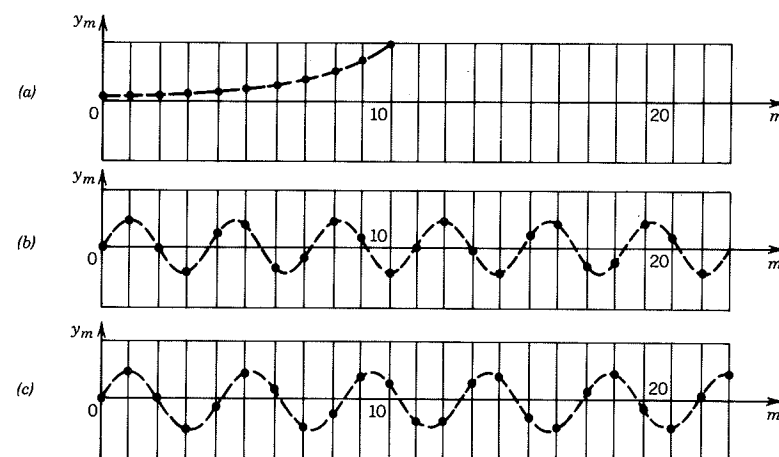


Figure 1.4-6 Examples of trajectories in periodic optical systems: (a) unstable trajectory ( $b > 1$ ); (b) stable and periodic trajectory ( $\varphi = 6\pi/11$ ; period = 11 stages); (c) stable but nonperiodic trajectory ( $\varphi = 1.5$ ).

retraces its path after  $s$  stages. This condition is satisfied if  $s\varphi = 2\pi q$ , where  $q$  is an integer. Thus the necessary and sufficient condition for a periodic trajectory is that  $\varphi/2\pi$  is a rational number  $q/s$ . If  $\varphi = 6\pi/11$ , for example, then  $\varphi/2\pi = \frac{3}{11}$  and the trajectory is periodic with period  $s = 11$  stages. This case is illustrated in Fig. 1.4-6(b).

#### Summary

A paraxial ray traveling through a cascade of identical unit optical systems, each with a ray-transfer matrix with elements  $(A, B, C, D)$  such that  $AD - BC = 1$ , follows a harmonic (and therefore bounded) trajectory if the condition  $|A + D|/2 \leq 1$ , called the **stability condition**, is satisfied. The position at the  $m$ th stage is then  $y_m = y_{\max} \sin(m\varphi + \varphi_0)$ ,  $m = 0, 1, 2, \dots$ , where  $\varphi = \cos^{-1}[(A + D)/2]$ . The constants  $y_{\max}$  and  $\varphi_0$  are determined from the initial positions  $y_0$  and  $y_1 = Ay_0 + B\theta_0$ , where  $\theta_0$  is the initial ray inclination. The ray angles are related to the positions by  $\theta_m = (y_{m+1} - Ay_m)/B$  and follow a harmonic function  $\theta_m = \theta_{\max} \sin(m\varphi + \varphi_1)$ . For the paraxial approximation to be valid,  $\theta_{\max} \ll 1$ . The ray trajectory is periodic with period  $s$  if  $\varphi/2\pi$  is a rational number  $q/s$ .

**EXAMPLE 1.4-1. A Sequence of Equally Spaced Identical Lenses.** A set of identical lenses of focal length  $f$  separated by distance  $d$ , as shown in Fig. 1.4-7, may be used to relay light between two locations. The unit system, a distance  $d$  of free space followed by a lens, has a ray-transfer matrix given by (1.4-11);  $A = 1$ ,  $B = d$ ,  $C = -1/f$ ,  $D = 1 - d/f$ . The parameter  $b = (A + D)/2 = 1 - d/2f$  and the determinant is unity. The condition for a stable ray trajectory,  $|b| \leq 1$  or  $-1 \leq b \leq 1$ , is therefore

$$0 \leq d \leq 4f, \quad (1.4-31)$$

so that the spacing between the lenses must be smaller than four times the focal length. Under this condition the positions of paraxial rays obey the harmonic function

$$y_m = y_{\max} \sin(m\varphi + \varphi_0), \quad \varphi = \cos^{-1}\left(1 - \frac{d}{2f}\right). \quad (1.4-32)$$

When  $d = 2f$ ,  $\varphi = \pi/2$  and  $\varphi/2\pi = \frac{1}{4}$ , so that the trajectory of an arbitrary ray is

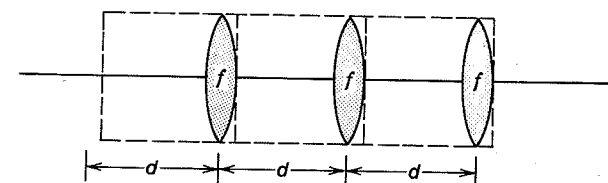


Figure 1.4-7 A periodic sequence of lenses.

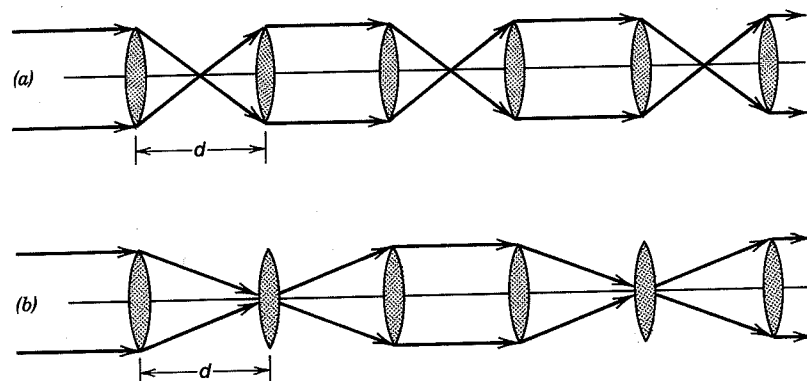


In recent years, as communications, computing, and printing, and the *Fundamentals of Optics* is a self-contained textbook that reflects the rapid changes in engineering and technology.

Featuring a wide range of applications and detailed analyses of the theories of wave optics and photonics, the interaction of light with matter, and the theory of lasers and their applications, the book is presented at a level of complexity that is suitable for building blocks for more advanced studies in optics and photonics, and sources and applications of light and acoustics, and optical communications and computer graphics, and other vital topics.

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Each chapter is highlighted with a list of exercises and example problems. Each chapter is included to give a governing approach to the subject, and the proper dimensional analysis of the linear system.

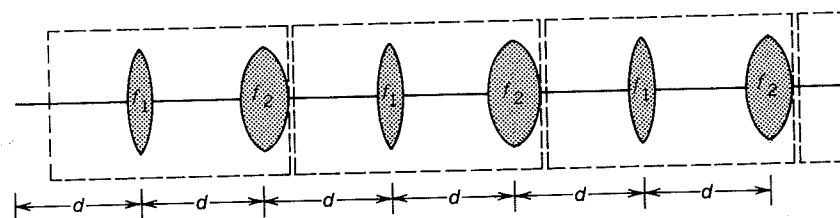


**Figure 1.4-8** Examples of stable ray trajectories in a periodic lens system: (a)  $d = 2f$ ; (b)  $d = f$ .

periodic with period equal to four stages. When  $d = f$ ,  $\varphi = \pi/3$  and  $\varphi/2\pi = \frac{1}{6}$ , so that the ray trajectory is periodic and retraces itself each six stages. These cases are illustrated in Fig. 1.4-8.

#### EXERCISE 1.4-6

**A Periodic Set of Pairs of Different Lenses.** Examine the trajectories of paraxial rays through a periodic system composed of a set of lenses with alternating focal lengths  $f_1$  and



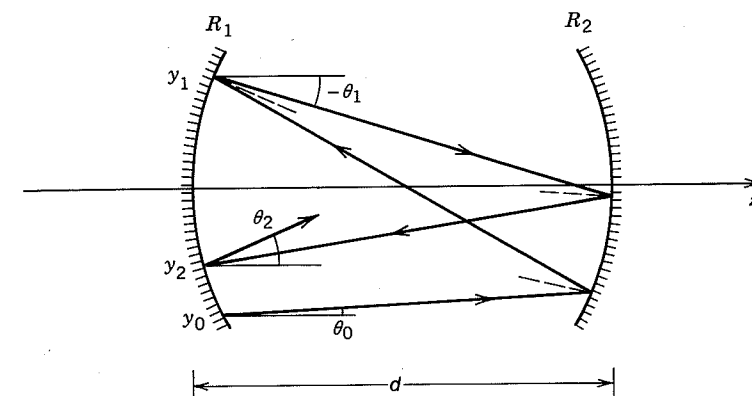
**Figure 1.4-9** A periodic sequence of lens pairs.

$f_2$  as shown in Fig. 1.4-9. Show that the ray trajectory is bounded (stable) if

$$0 \leq \left(1 - \frac{d}{2f_1}\right) \left(1 - \frac{d}{2f_2}\right) \leq 1. \quad (1.4-33)$$

#### EXERCISE 1.4-7

**An Optical Resonator.** Paraxial rays are reflected repeatedly between two spherical mirrors of radii  $R_1$  and  $R_2$  separated by a distance  $d$  (Fig. 1.4-10). Regarding this as a periodic system whose unit system is a single round trip between the mirrors, determine the condition of stability of the ray trajectory. Optical resonators will be studied in detail in Chap. 9.



**Figure 1.4-10** The optical resonator as a periodic optical system.

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