

# **Physics 408**

Applied Optics Laboratory

Department of Physics and Astronomy  
UBC

January 13, 2015



# Contents

<b>1</b>	<b>Physics 408 Laboratory Survival Guide: Rules and Guidelines</b>	<b>1</b>
<b>2</b>	<b>Safety Rules</b>	<b>9</b>
<b>3</b>	<b>Error Analysis</b>	<b>11</b>
<b>4</b>	<b>Fourier Optics</b>	<b>13</b>
4.1	Objective . . . . .	13
4.2	Introduction . . . . .	13
4.3	Alignment . . . . .	13
4.4	Mesh Filtering Experiment . . . . .	14
4.5	Character Recognition . . . . .	15
4.6	Dark-field Image . . . . .	16
4.7	Phase-Contrast . . . . .	16
4.8	Diffraction . . . . .	17
4.9	Useful Readings . . . . .	17
<b>5</b>	<b>The Michelson Interferometer</b>	<b>19</b>
5.1	Objective . . . . .	19
5.2	Introduction . . . . .	19
5.3	Finding the Fringes . . . . .	20
5.4	Calibration . . . . .	21
5.5	Index of Refraction of Air . . . . .	21
5.6	Determination Of Sodium Wavelengths . . . . .	22
5.7	The Position Of Zero Path Length Difference . . . . .	23
5.8	Fourier Transform Spectroscopy . . . . .	23
5.9	Useful Readings . . . . .	24
5.10	The Michelson Interferometer lab pre-reading . . . . .	24
<b>6</b>	<b>The Optical Cavity</b>	<b>25</b>
6.1	Objectives . . . . .	25
6.2	Introduction . . . . .	25
6.3	Mirror Reflectivity . . . . .	26
6.4	Beam Radius Measurement . . . . .	27
6.5	Optical Cavity Setup . . . . .	27
6.6	Piezo Calibration . . . . .	27
6.7	Cavity Observations . . . . .	28
6.8	Resonator Finesse . . . . .	29

6.9	Confocal Cavity . . . . .	29
6.10	Bonus experiment: EOM . . . . .	29
6.11	Useful Readings . . . . .	30
<b>7</b>	<b>The HeNe Laser</b>	<b>31</b>
7.1	Objective . . . . .	31
7.2	Introduction . . . . .	31
7.3	Transverse Laser Modes . . . . .	32
7.4	Measuring Mirror Curvatures and Stability Regions . . . . .	33
7.5	Polarization . . . . .	33
7.6	Spectral Output . . . . .	33
7.7	Mode Control and Beam Radius Measurements . . . . .	33
7.8	Additional Questions . . . . .	34
7.9	Useful Readings . . . . .	35
<b>A</b>	<b>CCD Camera Operation</b>	<b>37</b>
<b>B</b>	<b>Extended Fourier Analysis</b>	<b>39</b>
<b>C</b>	<b>The Monochromator and Photomultiplier</b>	<b>43</b>
<b>D</b>	<b>Modes of a Spherical Resonator</b>	<b>47</b>
<b>E</b>	<b>Resonator Theory</b>	<b>51</b>
<b>F</b>	<b>Images of Components for Cavity Lab</b>	<b>57</b>
<b>G</b>	<b>Using the Oscilloscope</b>	<b>65</b>
<b>H</b>	<b>Calibration Of The Interferometer</b>	<b>67</b>
<b>I</b>	<b>Fourier Transform Spectroscopy</b>	<b>69</b>
<b>J</b>	<b>Michelson Interferometer MATLAB Scripts</b>	<b>75</b>
<b>K</b>	<b>Image Formation</b>	<b>77</b>
<b>L</b>	<b>Concave and Convex Lenses</b>	<b>83</b>
<b>M</b>	<b>Phase-Contrast Imaging</b>	<b>85</b>
<b>N</b>	<b>Diffraction</b>	<b>89</b>



# Chapter 1

## Physics 408 Laboratory Survival Guide: Rules and Guidelines

Welcome to Physics 408 Laboratory!

Below you will find the rules and guidelines for the lab, including the marking breakdown for each component you are expected to complete for each lab module (total of four modules). You will be expected to complete an outline, introductory questions and the lab write-up for each module.

### **Warning:**

Because the course runs concurrently with the lab, you may be faced with learning and using completely new concepts in the lab BEFORE you have seen them in the course lecture. You may also be required to do calculations that you have little or no experience doing.

This situation is commonplace in research environments and industry (i.e. real life) - so this is a desirable difficulty that you will learn to overcome. To help you get through this trial-by-fire intact, pre-reading and resources are listed at the end of each lab.

### **Lab Overview:**

You will have four lab modules this term. You will have two or three weeks to complete each lab module (this will be finalized in the first week of class).

Following are some general rules and procedures for the labs:

- Absolutely no food or drink is allowed in the lab. You may, however, eat out in the main hallway.
- You are required to complete all 4 experiments and associated lab reports to pass the course.
- You must pass the lab component of the course to pass the course.

## 2CHAPTER 1. PHYSICS 408 LABORATORY SURVIVAL GUIDE: RULES AND GUIDELINES

- After completing a lab, your lab writeup is due at the beginning of the next lab period.
- You should buy two lab (yellow) notebooks at the bookstore. You will have one week to hand in your finished lab - due the next lab period after finishing that lab. While that notebook is being marked, you will use your second notebook for the next lab.
- You will work in groups of 2, and both partners must document separately all data and information. The schedules and the names of the section teams will be posted on the website.

Following is a general break down of what you should aim to complete during each lab period:

- At the beginning of the first lab period you will submit your outline for the current module and lab from the previous module. The only exception is the first week where you will only hand in an outline.
- In your first lab period you should focus on scanning through the lab and getting yourself familiar with the equipment. Do not feel like you need to be following the lab in the presented order, skip around the sections as you wish.
- In your second lab period, you should take some data and work through the identified difficult parts of the lab.
- In your final lab period, you should have attempted some of the analysis and be ready to retake any data that was not sufficient. Complete all necessary data and parts of the lab that require the use of the equipment.

### Marking:

Outline - 10%

Introductory Questions - 10%

Lab - 80%

**Late penalties** (We will be very strict about these penalties and there will be no exceptions):

- Labs are **due** at the beginning (not even 5 minutes late) of YOUR lab section when you are starting a new lab module. That means you typically have three weeks from the start of a lab to when you submit your lab.
- Labs **not** submitted at the beginning of YOUR lab section: **50% reduction**. You have ONE additional week to submit your lab without additional reduction. You can submit at any time during that week, so you can find time to complete the lab to your best ability.
- > 1 week late: The labs will **no** longer be accepted.

## Outline

You will be expected to complete a one page outline due at the beginning (not even 5 minutes late) of YOUR lab section of the first week of a lab module.

### Expectations:

You should print TWO copies of your outline, one to submit for comments and marks (this will be returned to you at the start of the second week of your module) and one to keep for use during the lab.

These outlines are designed to help you prepare for the lab and think about the experiments you will perform over the course of the module. It will also help you plan out how you will complete the lab in the given amount of time, what parts of the lab may be most challenging, and what data needs to be collected in the lab and what analysis can be done at home.

### Outline Guideline:

- Must be ONE page
- You should include a breakdown of the three lab periods with the tasks you plan to complete during each lab period.
- Specifically focusing on the data you need to gather in the lab, rather than the analysis that can be completed at home.
- Answer the following question: What part of the lab will take the longest to complete? What part of the lab take the least amount of time to complete? Why do you think so?

## Introductory Questions

During the first lab period for every module you will be handed 2-3 introductory questions. They will be handed out with 30 mins remaining in the period and **MUST BE SUBMITTED** at the end of the period.

The questions are randomized from a pool of questions and can be based on any part of the lab. They are simple qualitative, short answer questions. You will not be expected to do any calculations.

## Lab

### Expectations:

You are expected to write up all of your work in your lab book. This includes all data, plots, calculations and observations. You are expected to complete all tasks outlined in the lab manual and answer all of the questions. Your lab write up should be a good working record of your measurements, results, and calculations. The finished lab is not meant to be a formal writeup, but a neat copy of a working lab book, so that, say in 6 months, you could use that notebook to write up a formal report or journal article. You should spend enough time to produce a clear report that the marker can understand and follow easily. If mistakes are made, they should be neatly crossed out, not erased. Only the lab notebooks should be used (i.e., no loose leaf paper).

- At the top of each page, write the title, date and page number.
- Record carefully everything you're doing as you do it using subtitles such as Procedure, Apparatus, Results, Analysis or Discussion, and Conclusion.
- Document your work with schematic diagrams where necessary.
- All original data and observations should be entered directly into the lab book. Tables and graphs must be clearly titled and the axes labelled. There is no need to repeat anything in the manual but you should make notes of what you did, with diagrams, and anything unexpected.
- Provide a clear summary of your objective in each section.
- Answer all questions posed in the lab manual. Include units and uncertainties on all measurements and propagate units and uncertainties through all calculations. You can propagate the numbers and units separately if you so choose. Also, you can include just one example of a given class of calculation performed.
- Include uncertainties on all measurements as well as a justification.
- Include an error analysis of your results (i.e. propagate your errors through all calculations). In case you've forgotten how to do this look here: Error Propagation.
- Analysis: compare your results with expected results and analyze possible sources of error. A good comprehension of the theory will be needed for this part. Where possible, suggest improvements to the methods or apparatus used.
- Provide a conclusions section.
- If you perform work beyond the scope of the lab, bonus marks may be awarded.

#### **Additional Tips to Succeed in Lab:**

These labs are designed for you to engage like an experimentalist. You are expected to explore the content and discover how to observe the theories you will learn throughout the course. You should be asking yourself questions throughout the experiment and always be recording your observations.

#### **Questions to Ask Yourself:**

- Why?
- Does my answer make sense? If not, why?
- What could I do differently to get better results?
- Why is this important?
- What did I learn?

## Examples of Lab Notebooks:

Below are examples of a good and bad lab notebook with highlights of what makes them good and bad.

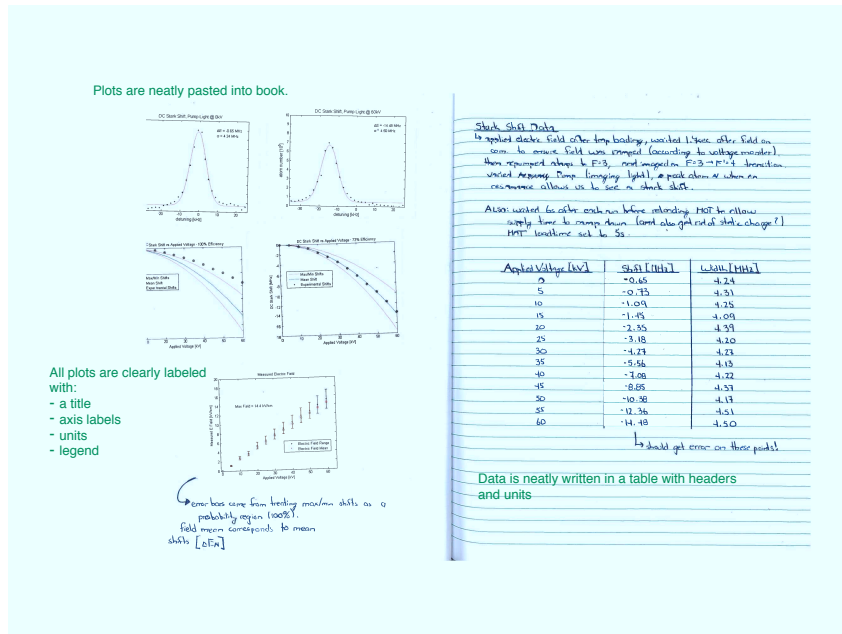


Figure 1.1: Example 1 of a Good lab notebook

## 6 CHAPTER 1. PHYSICS 408 LABORATORY SURVIVAL GUIDE: RULES AND GUIDELINES

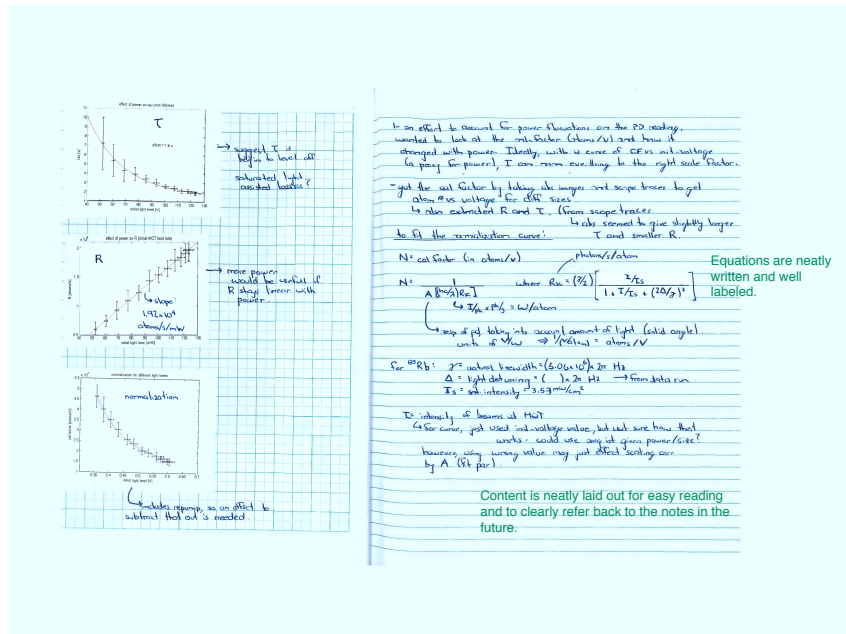


Figure 1.2: Example 2 of a Good lab notebook

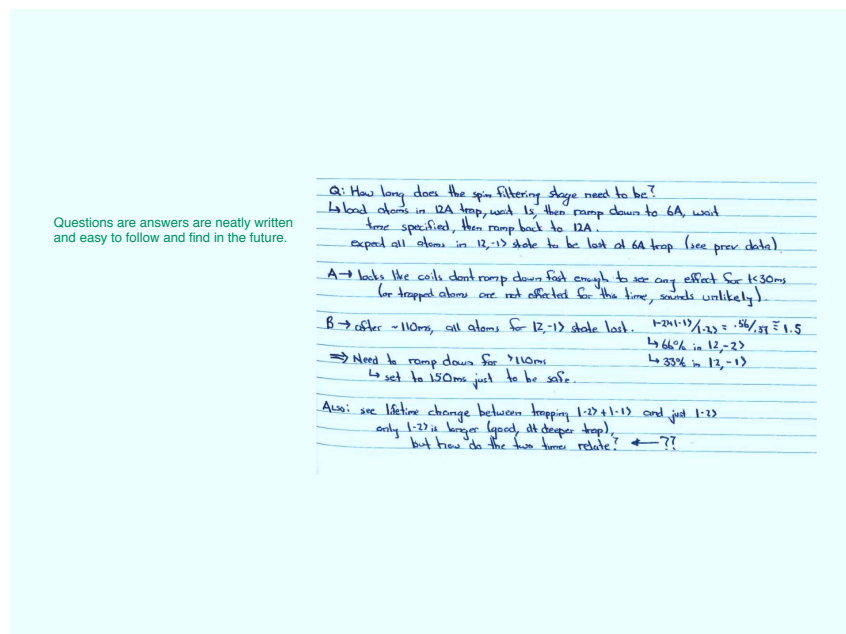


Figure 1.3: Example 3 of a Good lab notebook

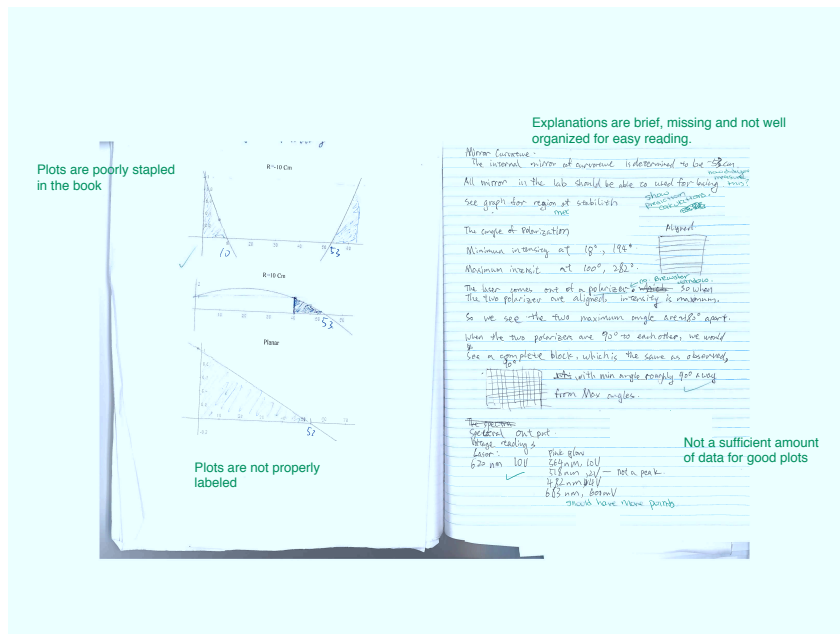


Figure 1.4: Example 1 of a Bad lab notebook

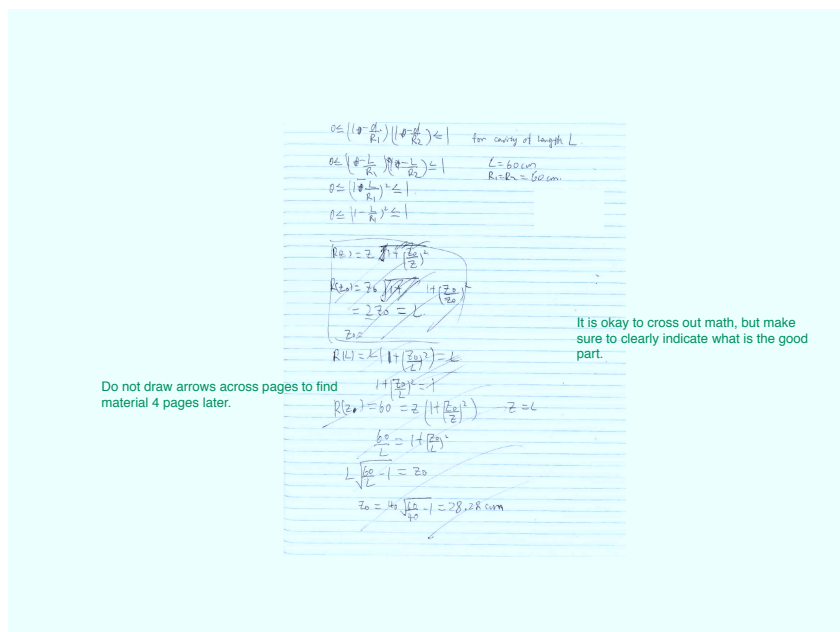


Figure 1.5: Example 2 of a Bad lab notebook





## Chapter 2

# Safety Rules

Please do not be apprehensive of this lab. If you are careful the danger involved in this lab is extremely minimal. However, if you fail to heed the following warnings bad things will happen.

### **Do Not Touch the High Voltage Electrodes in the HeNe lab**

There is a plastic casing surrounding the laser tube, there is no reason for you to remove this casing or to place your hands within the confines of it. People have died by mishandling laser power supplies. You won't, but there is no reason to test this theory.

### **Do Not Stare Into The LASER Beam**

This is not a high power laser beam; if for some reason the beam does impact near the area of your eyes your 'blink reflex' should be enough to protect you. This does not mean you should place your head/eye in the path of the beam to see where it is going. A small index card (provided) is a much better means of observing the path of the beam. It is also important to watch for stray reflections off of mirrors or other reflective surfaces. This means that any **watches, rings, or bracelets should be removed** before beginning this experiment. Laser goggles are provided.

### **Do Not Touch Any Optical Surface**

It does not take many impurities on the optical surfaces to prevent lasing action from occurring in the HeNe lab. Scratches, fingerprints, and even dust on the cavity elements can prevent a laser or a cavity from working.

### **Other Rules**

1. Absolutely **no food or drink is allowed in the lab rooms**. If you must eat, do so in the hallway.
2. **Do not move optics mounts or other hardware fixed to the bench with screws painted in red**. These are elements that are already in their proper place and moving them will make doing the lab impossible. If

you don't have a tool to loosen a screw, you probably shouldn't be trying to move that component. If you do move an optical element held with red screws or suspect that such an element is not in its proper place, please immediately contact the lab TA or professor to help you reset this element - obtaining good data and completing the lab may depend on it.

## Chapter 3

# Error Analysis

Whenever you are trying to observe and quantify some physical phenomenon and relate it to a model or otherwise quantitative description of said phenomenon, it is absolutely essential that you record with your measurements the uncertainty in those measurements. You can then include the uncertainties (and propagate the error) in any comparison you make with a theoretical prediction or a theoretical model which relates the measurements you have made. This propagation of the uncertainties will allow you to replace a qualitative (and meaningless) statement like “our results agree pretty well with theory” with a statement about just how well your results agree or disagree with theory, quantitatively. For your lab reports, we expect that you will include uncertainties in all measurements and use them in all comparisons with theory.

There are numerous resources available on error propagation and error analysis. The following are two that you might find useful:

<http://lectureonline.cl.msu.edu/~mmp/labs/error/e2.htm>

[http://teacher.pas.rochester.edu/PHY\\_LABS/AppendixB/AppendixB.html](http://teacher.pas.rochester.edu/PHY_LABS/AppendixB/AppendixB.html)



## Chapter 4

# Fourier Optics

Note: The computer in this lab should be run in the session type KDE (don't use default session type). This can be changed at login.

### 4.1 Objective

In this lab you will explore Fourier transforms and diffraction patterns using optics. Specific objectives are:

1. To explore and discover Fourier transformations using a laser and optics.
2. To explore phase contrast imaging when certain spatial frequencies experience an intensity or a phase change relative to other spatial frequencies.
3. To investigate diffraction from a slit.

### 4.2 Introduction

### 4.3 Alignment

You will need to carefully align the laser and spatial filter and collimating lens to generate a large and defect free Gaussian beam. Other optics and objects (for example, lenses) must be placed along the beam path (the rail) in order to perform the different experiments in the lab. Figure ?? illustrates the *approximate* positions of the major optical components on the 4m optical bench.

The spatial filter is comprised of a microscope objective and a pinhole mounted on translation stages and should be placed just after the laser. The microscope objective lens focusses the laser through the tiny ( $\sim 25$  to  $50\ \mu\text{m}$ ) pinhole, and this produces a divergent, coherent beam with a clean spatial profile (i.e. no transverse spatial structure). Refer to Fig. ??.

1. Align the spatial filter. Why is the spatial filter important for this lab?
2. Put together an imaging system using the lens labeled as the 'Fourier Transform lens'. What is the best object to use in order to find the object plane? Why? Use this object to find the object plane. Hint: it may be useful to magnify your image onto the screen on the other rail.

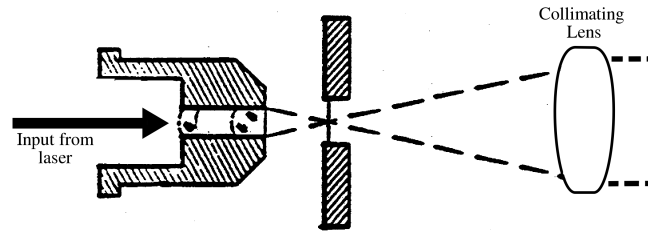


Figure 4.1: A spatial filter consists of a microscope objective, which can be adjusted to focus a laser beam through a small pinhole.

3. What is the best object to use in order to find the Fourier transform plane? Use this object to find the Fourier transform plane.
4. Create a sketch in your lab book (similar to Fig. ??) showing the location of each of the lenses and planes in your setup.

## 4.4 Mesh Filtering Experiment

In this section, use the imaging setup to generate a magnified image of the mesh aperture on the screen.

1. From the number of wires per cm given on the mesh aperture, and the spacing of the image wires on the screen, estimate the magnification of this system.
2. Does this magnification agree with the thin lens formula? Include a picture of the magnified image. Be sure to indicate the scale.
3. Magnify the Fourier transform plane onto the screen and take a picture.
4. What do the bright spots in the Fourier transform plane represent?
5. Rotate the mesh around in the mount and lift it up and down. How does moving the mesh change the magnified image and the Fourier transform? Why do some types of movement change the image but appear not to change the Fourier transform of the image? What might be happening?
6. Using the Fourier transform image, calculate the spatial frequencies that are present. (Please be careful with units.)
7. What wire spacing do these spatial frequencies correspond to this correspond to?

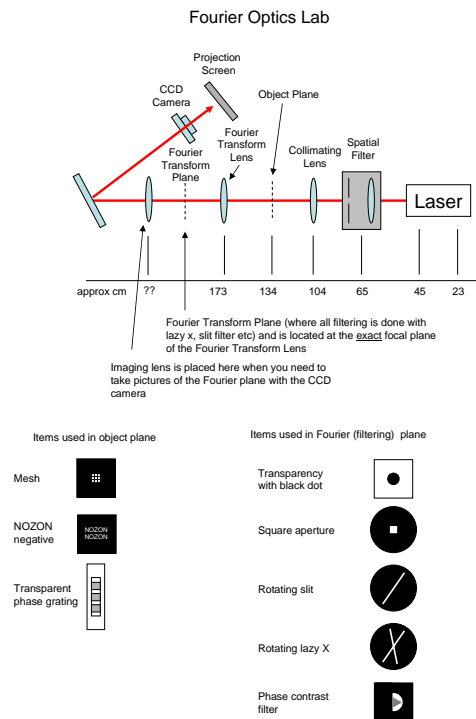


Figure 4.2: Experimental arrangement showing the *approximate* positions of the major optical components on the 4m optical bench as well as the various object and filter components used in this lab. The laser is mounted on one end, the plane mirror on the other. Position the mirror so that the beam illuminates a screen on the secondary rail.

8. How does this compare to what you expect?
9. Using the mesh object, how can you produce only vertical lines? Explain how and why this is happening. Provide a picture.
10. Try producing lines at  $45^\circ$  and horizontal. If you are able to produce these lines, explain how you did it and explanation of why its works. Include a picture with your explanation.

## 4.5 Character Recognition

Here you will use spatial filtering techniques to choose only the “N”s from the NOZON aperture.

1. Using the NOZON aperture, obtain the Fourier transform and carefully look at the pattern of the spatial frequencies. Include a picture and de-

scription of your observations. Why is this pattern more complex than the mesh?

2. Based on the Fourier transform and your knowledge of spatial filtering, can you filter out all the letters except the N's from the original object? How does this filtering work? Include a picture and explanation of what you did and why it worked.

## 4.6 Dark-field Image

In this task you will explore what a “dark-field image” is and how it can be obtained.

1. What is meant by a “dark-field image”? What aperture can you use to produce a dark field image?
2. Which object will give the best visibility of the “dark-field image” effect? Justify your answer.
3. Which aperture will you use in the Fourier transform plane to produce the dark field image? Describe and explain your observations and take a picture for your lab book.

## 4.7 Phase-Contrast

In this task you will alter the phase of selected spatial frequencies using a “phase plate” (in this case, the phase plate is just a plastic wedge that is placed into the Fourier transform). You will use the grating aperture in the object plane. This so called *phase contrast imaging* maps phase variations to intensity variations. If you can't find the aperture, ask a TA.

1. Explain how the grating causes a spatially dependent phase in the original light wave.
2. Insert the vertical razor blade with a wedge of clear plastic taped to it into the Fourier Transform plane. How does this “phase plate” allow you to modify the phase of the DC component while leaving all (most) of the other components unaffected?
3. You should see two effects in your image of the phase grating. Take a picture and identify and explain the two effects you observe.
4. How can you prove you are truly observing the grating? Explain your reasoning.
5. Using the phase-contrast image of the phase grating, compute the magnification of your imaging system. How does this compare to your previous measurement with the mesh?
6. Can you use this technique to measure the thickness variation in an object? Why or why not? Try it using the DC block aperture as your object. Record the image.



## 4.8 Diffraction

You will investigate the two (Fraunhofer and Fresnel) diffraction regimes by studying the diffraction pattern from a slit aperture of variable width. The diffraction pattern observed can be characterized by a dimensionless parameter,  $\Delta v$ , defined as;

$$\Delta v = w \left( \frac{2}{R\lambda} \right)^{1/2} = \left( \frac{R_0}{R} \right)^{1/2} \quad (4.1)$$

where  $w$  is the width of the slit, and  $R$  is the slit-screen distance. Fresnel diffraction occurs for  $\Delta v \geq 1$ . Fraunhofer diffraction (the so called far-field approximation) is characterized by  $\Delta v \leq 1$ .

1. What modifications to the setup do you need to make in this part of the experiment?
2. Insert the slit and look at the diffraction pattern for a very small slit width and a very large slit width. Describe and explain the pattern you see at these two different slit widths.
3. Record the image of the diffraction patterns for at least five slit widths. Be sure to include some patterns in both regimes, and near the crossover between the regimes.
4. Plot the diffraction pattern intensity as a function of position along an axis perpendicular to the slit axis for each image in (3).
5. For each set of data, plot the Fraunhofer and Fresnel patterns (predicted by the value of  $\Delta v$ ) against your data and compare the results. For each  $\Delta v$  which theory does a better job of modeling your data? Justify and explain your reasoning.

### Final Question

What was the most challenging part of this lab? Is it what you thought it would be (i.e., what you wrote in your outline)? Comment (i.e., write more than yes or no).

## 4.9 Useful Readings

- Section 4.1 Propagation of Light in Free Space
- Section 4.2 Optical Fourier Transform
- Section 4.3 Diffraction of Light
- Section 4.4 Image Formation (but not part D on Near-Field Imaging)
- Appendices: A,B,K,L,M,N



## Chapter 5

# The Michelson Interferometer

### 5.1 Objective

In this lab you will explore interference patterns produced by light from three different sources (a HeNe laser, a sodium lamp and a white light source). Specific objectives are:

1. To observe and modify the shape of interference patterns.
2. To use the information encoded in the interference pattern to determine information about the source emission wavelengths or the differences in the beam paths.
3. To perform Fourier transform spectroscopy to determine the wavelengths emitted by the electromagnetic sources.

### 5.2 Introduction

A well known interferometer, operating on the principle of division-of-amplitude, is the Michelson interferometer. The incident light beam is divided into two parts by means of a beam-splitter. The divided beams traverse different paths and then are recombined. Depending on the difference in optical path lengths of the two beams, the recombined beams may be  $180^\circ$  out of phase, producing destructive interference, or in phase, producing constructive interference. Observation of the resulting interference fringes while changing the optical path difference allows one to determine wavelengths, wavelength differences, and/or small differences between the two optical paths. When the path length difference between the two arms is zero, this is referred to as the “zero path length” (ZPL). Fig. ?? illustrates the physical arrangement of the Michelson interferometer.

Use of this interferometer as a spectrometer to measure the power spectrum of an emitting source became feasible with the advent of high speed, large storage capacity, computers which could calculate the Fourier transform of the output irradiance of the interferometer. You will use this instrument to realize such a Fourier transform spectrometer.

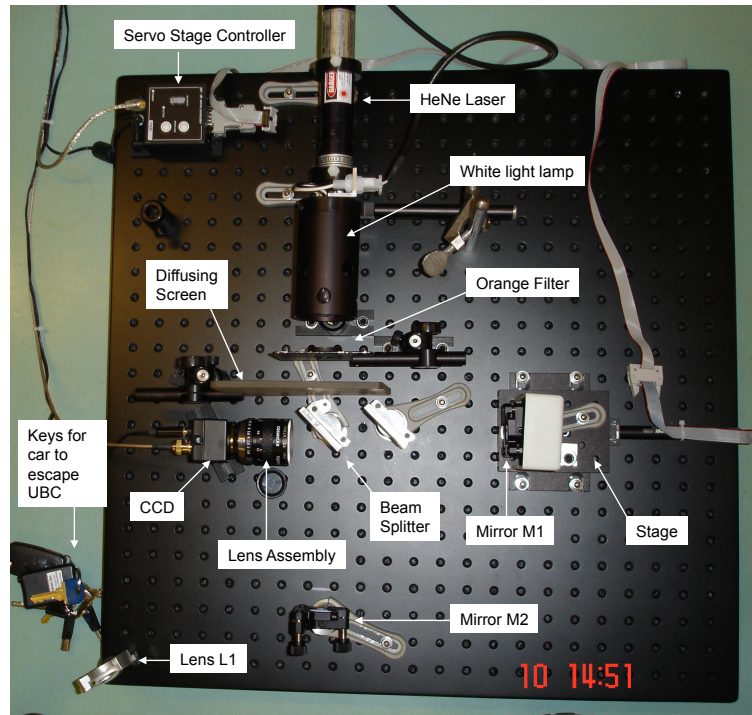


Figure 5.1: The Michelson Interferometer. Note, the sodium lamp is not pictured

### 5.3 Finding the Fringes

Use the Sodium lamp in this section of the experiment. Position the stage near the last recorded ZPL (you can find this number on the clipboard near the experiment), and align the M2 mirror such that it is perpendicular to the M1 mirror (at which point you should see fringes on the camera).

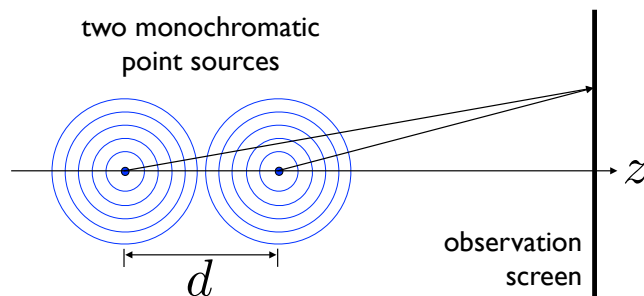


Figure 5.2: Two monochromatic point sources separated by a distance  $d$  on the  $z$ -axis create an interference pattern on a screen

1. What type of fringe patterns can you make? Take a picture of each fringe pattern and explain what you did to produce each type.

You can compute the fringe pattern for the Michelson interferometer by modeling the system as two point sources separated by some distance (see Fig. ??).

1. Justify why this is a good model for the interferometer you are using in the lab.
2. Write down the electric field amplitudes ( $U_1$  and  $U_2$ ) from two spherical wave sources and derive an expression for the total intensity  $I = |U_1 + U_2|^2$  as seen on the observation screen.
3. Using this expression, describe how the intensity on axis varies with  $d$ .
4. Describe what happens to the interference pattern if you change  $d$  along the  $z$ -axis. Why might knowing this information be useful in this lab?

## 5.4 Calibration

In many applications of the interferometer, you will scan the path length of one arm by moving a translation stage over a set distance. To do this, you input both a distance and speed into a MATLAB program. When you capture data from the camera, the program records the intensity of a point on the CCD at a rate of 30 frames per second. This means the distance the stage will have moved per frame is dependent on the speed you set. For example, for a speed of  $1 \mu\text{m/s}$  the stage will move  $\frac{1 \mu\text{m/s}}{30 \text{ frames/s}} = \frac{33 \text{ nm}}{\text{frame}}$ . You should use the HeNe laser in this section.

1. Do you think the stage will move **exactly** the distance that you set? Why or why not?
2. What does it mean to calibrate the translation stage? What two pieces of information do you need to find the calibration factor?
3. What distance does the mirror need to move such that one fringe passes a reference point on the CCD camera?
4. Calibrate the interferometer at two different speeds. You should pick  $1 \mu\text{m/s}$  to be one of the speeds.

## 5.5 Index of Refraction of Air

The interferometer can be used to characterize extremely small changes in the optical path length differences between the two arms. You can use this sensitivity to even measure the phase shift induced by the propagation of light through air and, by extension, the index of refraction of air. You should use the HeNe laser in this section.

1. How many times does the beam traverse the cell in the interferometer? Compute the total optical path length difference for light at 632.8 nm HeNe with and without air in a vacuum cell 10 cm in length. What is the expected number of fringes that will pass as air is leaked into a cell of this length initially under a perfect vacuum?
2. Measure the index of refraction of air. Hint: You should use the `vid_capture` program.
3. Would you have seen the same number of fringes if you had used the Na lamp instead of the HeNe laser for this measurement? Why or why not?
4. What is your value for the index of refraction of air for 632.8 nm light? Does this agree with the “accepted” value? What are your sources of error?
5. Could you use this apparatus as a barometer or thermometer?

## 5.6 Determination Of Sodium Wavelengths

Excited sodium atoms primarily emit light at two wavelengths,  $\lambda_1$  and  $\lambda_2$ , also known as the D1 and D2 lines, separated by some  $\Delta\lambda$ , where  $\Delta\lambda \ll \lambda_1, \lambda_2$ . The fringes from the D1 and D2 lines always overlap at the position of zero path length difference. However, if the stage is moved away from this position, then a new position will be reached (at a distance  $d$ ) where the bright fringes of one line fall on the dark fringes of the other. The resulting fringe pattern is then very indistinct and the fringes are said to have low visibility. This particular distance is

$$2d = (N + 1)\lambda_1 = N\lambda_2 \quad (\lambda_1 < \lambda_2) \quad (5.1)$$

where  $N$  is an integer. The wavelength difference is, therefore;

$$\Delta\lambda = \lambda_2 - \lambda_1 = 2d \left( \frac{1}{N} - \frac{1}{N + 1} \right) = \frac{\lambda_1 \lambda_2}{2d} \quad (5.2)$$

If  $\lambda_1$  and  $\lambda_2$  differ by very little, it is sufficient to replace their product by  $\lambda^2$ , where  $\lambda$  is the average of the two.

1. What speed and span would be reasonable to use to measure the wavelength and the wavelength difference? Why?
2. Determine the average wavelength of the two Sodium lines. Hint: You will need to use similar method as the calibration task.
3. Measure the difference in the wavelength between the D1 and D2 line (i.e.,  $\Delta\lambda$ ).
4. From the data you have obtained, compute the wavelength of the D1 and D2 lines and compare with the known values.

## 5.7 The Position Of Zero Path Length Difference

Locating the position of zero path length difference is crucial in order to observe a fringe pattern from white light. Additionally, you will get the best results in the Fourier Transform Spectroscopy section when your data is collected about the position of zero path length difference.

1. In your own words, describe what is meant by the term “coherence length”. How might you measure the coherence length of a certain source using the interferometer in the lab?
2. Measure the coherence length of the sodium lamp. Once you have done this, position the translation stage in the position that gives a fringe pattern with the largest visibility.
3. Replace the sodium lamp with the white light source, and insert the orange filter. Measure the coherence length of the white light source with the orange filter. Once you have done this, position the translation stage in the position that gives a fringe pattern with the largest visibility.
4. Remove the orange filter and measure the coherence length of just the white light source.
5. Where should the stage be placed in order for the two arms of interferometer to have a zero path length difference?

Given your observations, answer the following questions:

1. Assuming that your interferometer is aligned and at the ZPL, describe how you would need to move the mirrors in order to achieve the ZPL if a single 1 mm thick glass plate (index of refraction 1.52) was added to one of the interferometer arms and you were using monochromatic light at 632 nm.
2. How does your answer change if you are using monochromatic light at 589 nm but the index of refraction at this wavelength is also 1.52?
3. It turns out glass is actually dispersive. What does this mean?
4. Why is there a compensator plate in the experimental set up? Is it necessary for all of the light sources?
5. In a sentence or two explain what is meant conceptually by the zero path length difference. Does being at the position of zero path length difference mean that the physical path length of the two arms is the same?

## 5.8 Fourier Transform Spectroscopy

In this section, you will measure the spectrum for each of the light sources in the lab using Fourier transform spectroscopy. Set the translation stage such that you are at the ZPL. All data should be taken around the ZPL.

1. Take at least one spectrum for each of the white light, white light with orange filter, sodium lamp, and the HeNe laser.
2. Take the Fourier transform of each spectrum.
3. Compare the transform to what you expected. Discuss possible causes of any discrepancies you may see.
4. Does the 589-nm interference filter have a Gaussian transmission function? Why or why not?
5. What is the measured width of the spectral peak of the HeNe output in nm? The linewidth of the HeNe laser is well below 100 MHz. Can you resolve the HeNe linewidth from your measurements? Why or why not?
6. Explain what would happen to your data if you made the **span** of your scan larger or smaller. Explain what would happen to your data if you made the **speed** of your scan larger or smaller.
7. If you wanted to retake your HeNe data in order to resolve the linewidth, what changes would you make to the measurement?

### Final Question

What was the most challenging part of this lab? Is it what you thought it would be (i.e., what you wrote in your outline)? Comment (i.e., write more than yes or no).

## 5.9 Useful Readings

### 5.10 The Michelson Interferometer lab pre-reading

- Section 2.2 Monochromatic waves (Plane Wave, Spherical Wave, The Paraboloidal Wave)
- Section 2.5 Interference
- Section 11.2 Interference of Partially Coherent Light, including part A. Interference of Two Partially Coherent Waves and part B. Interference and Temporal Coherence, and Fourier-Transform Spectroscopy
- Appendices: H,I,J



## Chapter 6

# The Optical Cavity

### 6.1 Objectives

In this experiment the goal is to explore the spatial and temporal characteristics of the transverse and longitudinal modes a two-mirror optical resonator. This resonator is similar to the HeNe laser resonator (that you have or will investigate in another lab) except that there is no gain element in the cavity and it is therefore a “passive” resonator. Specific objectives are:

1. Explore the concepts of resonance, spatial mode matching, and resonator stability using the  $\text{TEM}_{00}$  transverse mode of a Helium-Neon laser and an external two-mirror optical cavity.
2. Investigate the dependence of the cavity finesse, linewidth and free spectral range on cavity length.
3. Use the cavity to measure the linewidth of the HeNe laser source.

### 6.2 Introduction

One of the most common methods used in the characterization of laser light involves sending that light into an external optical resonator (i.e. cavity). Suppose, for the moment, that the laser light is purely monochromatic and has a  $\text{TEM}_{00}$  beam profile. The laser light will pass into the external cavity if the cavity length were stabilized and if the frequency of the laser light were resonant with the frequency of one of the modes of the cavity. For ideal coupling of the light into the cavity, the transverse mode profile of the laser light should match the transverse profile of the cavity mode with which it is resonant. So, if the laser light propagates in a  $\text{TEM}_{00}$  mode (by which we mean the fundamental transverse mode of the laser resonator), then we would want it to match the fundamental transverse mode ( $\text{TEM}_{00}$ ) of the external cavity. Such mode matching does not occur automatically since the fundamental mode of the external cavity will have its own beam shape (i.e. spatially varying beam width and radius of curvature) set by the cavity mirrors, and this will be independent of the beam shape of the laser. To mode match the laser  $\text{TEM}_{00}$  mode to that of the external cavity mode, lenses (or, in this case, one lens) must be used to

shape the incoming beam so that the parameters of the input beam match those of the resonant cavity mode.

As usual, the experimental situation is more complex than the ideal case. If the laser beam is not perfectly aligned and mode-matched to the external cavity, the input beam will partially couple to many different transverse modes of the cavity as the cavity length is changed and these modes come into resonance with the frequency of the laser output (the mode of the input beam can be described as a superposition of external cavity eigenmodes- such as the Hermite-Gaussian modes). The light that exits the cavity will resemble whatever cavity modes happen to be excited rather than resembling the spatial profile of the original input beam. Since the cavity length in this lab is not stabilized, microscopic vibrations of the optical bench will change its length and the coupling of the laser beam to the longitudinal and transverse cavity modes will change with time, and the light patterns that are observed exiting the cavity will fluctuate with time, revealing a time-dependent coupling of the laser beam to many different high-order Hermite-Gaussian modes. Figure ?? shows a schematic of the setup you will use in this lab. The cavity consists of two mirrors: one flat, and one with a radius of curvature of 30 cm. The output of the HeNe laser has a beam diameter of 1.2 mm.

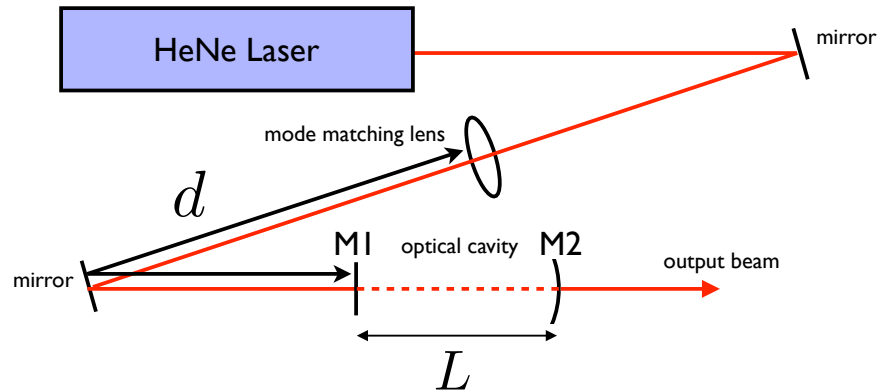


Figure 6.1: Cavity setup. The length of the cavity is the distance between M1 and M2 ( $L$ ), and the distance from the mode matching lens to the input coupler of the cavity (M1) is denoted  $d$ .

### 6.3 Mirror Reflectivity

1. Measure the reflectivity and transmission of both cavity mirrors. Do your reflectivity and transmission coefficients add to unity? Should they?
2. Calculate the finesse that you expect your cavity to have.
3. Given this finesse, for a cavity with length  $L=15$  cm, what do you expect the cavity linewidth and free spectral range to be? Express your answer in MHz and nm. Which do you think is a better unit of measure in this context?

4. What would happen to the cavity characteristics if the low reflectivity sides of the mirror faced each other?

## 6.4 Beam Radius Measurement

There are three lenses available to optimally couple (that is, mode match) the light into the  $\text{TEM}_{00}$  mode of the optical cavity. Unfortunately, none of them are the exact lens you require. For this section, we recommend that you choose the 50 cm focal length lens.

1. Sketch the beam shape and phase fronts of the cavity mode. Where is the focus of this beam? Hint: What boundary condition applies to the eigen-modes of the cavity?
2. At what location should the beam come to a focus in order to best mode match into the cavity?
3. Measure the minimum beam width (i.e. the beam waist) at the focus using a knife-edge measurement (See the write-up provided on the lab webpage that discusses a knife-edge measurement).
4. What beam waist should you have in order to best mode match into the cavity? Hint: Your answer might depend on the length of your cavity. Be sure to specify what length (or lengths) you have decided to use.
5. Given your answer for the ideal beam waist, what would be the ideal focal length lens to use? How far from the M1 mirror should it be placed?

## 6.5 Optical Cavity Setup

In this section, you will align the optical cavity (see Appendix REF and the video on the lab website for details on the alignment). As you setup the cavity, answer the following questions:

1. When only one mirror (M1) is placed in the beam path, what effect does it have on the transmitted light.
2. When you place the second mirror (M2) in the setup, what is the net effect on the light transmitted through both mirrors? Does it depend on whether or not the cavity is correctly aligned?

When you think you have the cavity aligned correctly (the first time), get a TA to check.

## 6.6 Piezo Calibration

Fine adjustments of the length of the cavity are produced by applying a voltage to the piezoelectric actuator (“piezo” for short) behind the M2 mirror. Voltage is supplied to the piezo by a high voltage supply, which has a manual adjustment knob and an external input. The external input is connected to a ramp generator. Note that the high voltage supply has a gain of 10x on the external input

when the voltage range is set to 100. The input multiplier changes depending on the voltage limit indicated by the green LED. The distance the actuator moves given a certain voltage change can be found on the actuator datasheet; however, each actuator is slightly different and you need to verify the response of your piezo.

1. What input signal (from the function generator) should you use to scan the piezo?
2. Use the function generator to apply a varying voltage to the piezo. Set the voltage such that you see a periodic pattern repeat three or four times. Take a picture of the cavity transmission (from the oscilloscope), and point out the periodic pattern that repeats.
3. What is the specification for the ThorLabs piezo you are using?
4. Find (experimentally) the calibration of the piezo (in units of  $\mu\text{m}/\text{V}$ ). Does it agree with the specification?

## 6.7 Cavity Observations

You can also use the CCD camera to observe the transmitted beam pattern while slowly scanning the piezo voltage by hand. It's helpful to set up both the camera and the photodiode such that you can easily switch between which one you direct the transmitted cavity light. For both a long and short cavity:

1. Do your best to optimize the transmission signal (that is, optimize the power in the fundamental cavity mode), and take a picture of the transmission pattern from the oscilloscope.
2. Use the CCD camera to take pictures of the transmitted cavity mode profiles. Indicate what kind of transverse mode it is, and specify what the spatial symmetry of the mode is.

Given your observations, answer the following questions:

1. Does the long or short cavity have more visible transverse modes? Why? Hint: What is the size of the beam at the M2 mirror? You can determine this both experimentally (i.e., look at it) and theoretically (i.e., calculate the beam radius at the position of M2 given what you know about the cavity).
2. Why do you see multiple peaks that repeat periodically, rather than just one? Which cavity mode likely corresponds to the largest transmission peak?
3. Why do different transverse modes occur at different cavity lengths?

## 6.8 Resonator Finesse

In this part of the lab, you will determine the finesse of your cavity by measuring the free spectral range of the cavity and the width of the transmission peaks.

1. Measure the free spectral range, linewidth and finesse of your cavity as a function of cavity length. Be very clear about the units of your measurements. Hint: The free spectral range and linewidth of the cavity should be on the order of 10 to 100s of MHz.
2. Plot the cavity finesse and cavity linewidth as a function of cavity length. On the same figure, plot the expected cavity finesse (you found this earlier) and the expected cavity linewidth (given the expected finesse). Do your results agree or disagree with what you expect?
3. From these results, what can you say about the spectral linewidth of the HeNe laser?
4. What is the  $Q$ -factor of your cavity?
5. Consider a tuning fork oscillating at 440 Hz (this is what musicians often use to tune their instruments). How long would the tuning fork ring given it had the same  $Q$ -factor as your optical cavity? Is this reasonable?

## 6.9 Confocal Cavity

1. For what range of lengths is the cavity stable?
2. What is the cavity distance  $L$  where the geometric factors drop out (i.e.  $g_1 g_2 = 0$ ) and you can see the modes becoming degenerate?
3. Align your cavity near this length, and slowly vary the cavity length using the M1 translation stage. Record the transmission spectrum as you approach the length where the modes become degenerate.
4. What happens to the transmitted power as you move the cavity length beyond the length over which it is stable?

## 6.10 Bonus experiment: EOM

Do not attempt this part until you have completed the rest of the lab. The alignment on this part is challenging, and you will need to realign the system from scratch once you insert the electro-optic modulator (EOM), so please notify the TA or Prof. that you wish to try it. Use the electro-optic modulator to induce sidebands on the laser light and measure the exact modulation frequency of your driver using the optical cavity as an optical spectrum analyzer.

### Final Question

What was the most challenging part of this lab? Is it what you thought it would be (i.e., what you wrote in your outline)? Comment (i.e., write more than yes or no).

## 6.11 Useful Readings

- Section 1.4 Matrix Optics [A,B,C,D], Condition for Periodic Trajectory (8 pages)
- Read the last subsection of Section 2.5 Interference, this part discusses the Interference of an Infinite Number of Waves of Progressively Smaller Amplitudes and Equal Phase Differences
- Section 3.1 The Gaussian Beam
- Section 3.2 Transmission through Optical Components (Beam Shaping and Beam Focusing)
- Section 3.3 Hermite-Gaussian Beams
- Section 10.1 Planar-Mirror Resonators
- Section 10.2 Spherical-Mirror Resonators (including parts A-D)
- Appendices: A,D,E,F,G

## Chapter 7

# The HeNe Laser

Note: you will need to adjust the horizontal and vertical tip and tilt of the external mirror in the lab but please do not adjust the height of the mirror or remove the mirror post from the post holder for any reason. Any alignment of the mirror should be done with the adjustment knobs with the mirror mount left in place. If you suspect that the mirror mount height has been changed, please contact a TA to correct this problem.

### 7.1 Objective

In this lab you will construct a HeNe laser. By observing and investigating its fundamental properties and operating conditions, you will learn how a laser works and what determines its output characteristics. Specific objectives are:

1. Investigate the dependence of the cavity stability on the radius of curvature of the cavity mirrors
2. Observe and characterize properties of the laser output (for example, the transverse spatial mode, spectrum and polarization).
3. Measure the beam radius at different locations inside and outside the cavity and compare with the expected size given the chosen cavity length and mirrors.

### 7.2 Introduction

The helium-neon (HeNe) laser was among the first lasers ever constructed, and has since proved to be among the most popular. In general, a laser consists of two basic elements: (1) a gain medium to provide optical gain and (2) an optical element (here a two-mirror resonator) to send the light emitted by the gain medium back into the gain medium thus providing optical feedback.

In this lab, you will build a laser using a gas mixture of helium-neon as the gain medium. A high-voltage power supply is used to supply the energy by exciting the gas mixture confined in a glass tube. These atoms exist in the gain medium, which is a glass tube filled with a helium-neon gas mixture. The optical resonator is constructed using one fixed, very high reflectivity mirror and

a second adjustable *output coupler* mirror. This mirror allows a small fraction of the photons in the laser to exit the resonator (normally around 1%). A schematic of the laser cavity is shown in Fig. ??.

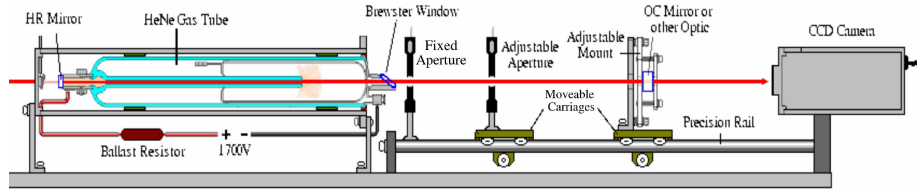


Figure 7.1: Open Frame HeNe Laser cavity. This is a schematic of the HeNe laser cavity that you will align in this lab.

The laser resonator you will build is capable of supporting more than one transverse spatial mode and more than one longitudinal mode. Each of these modes has associated with it a specific optical frequency and intensity distribution and only occur for certain configurations of mirror curvatures and separation.

The transverse mode of the laser refers to how the laser spot looks and is determined by the electric field distribution of the laser mode. These modes are labeled by  $TEM_{lm}$  (TEM stands for, not surprisingly, Transverse Electromagnetic), where  $l$  and  $m$  correspond to the number of electric field nodes along the  $x$  and  $y$  axis respectively.

It is typical in gas lasers such as the HeNe for the gain to be concentrated near the axis of the tube. As a consequence, any modes that have an electric field maxima on this axis tend to extract the most energy from the gain medium and thus dominate. However, if a small object (such as a thin wire) is placed such that it impedes the gain at the axis center, then only those modes with a node (zero) occurring at the location of the object will lase. Thus by translating the object laterally across the profile of the beam it is possible to change and control the spatial mode generated by the laser.

### 7.3 Transverse Laser Modes

For this section, use the planar mirror as the output coupler mirror, and place the mirror such that the cavity length is  $< 50$  cm. Use the fixed apperture to help align the fluorescence from the HeNe tube back onto itself. Next, make small adjustments to the  $x$  and  $y$  controls until you see a bright red glow on the mirrors. Your laser is lasing!

1. Continue to adjust the mirror, and look at the mode profile on a camera. Does the pattern change?
2. Capture some of these patterns and print them in your lab book. For each, explain their symmetry and label their mode structure. What is the highest order mode you could see?
3. With the laser lasing in any mode that is not the  $TEM_{00}$  mode, slowly open and close the iris near the Brewster window. What happens? Explain why.



4. Why do you usually see the laser has only one transverse spatial mode at a time?

## 7.4 Measuring Mirror Curvatures and Stability Regions

1. Use the planar mirror ( $R = \infty$ ) to determine the ROC of the fixed cavity mirror.
2. Measure the stability regions of the laser using the curved output coupler mirror. How does your measured region of stability compare with what you expect (given the information gathered in the previous step)? Use your measurements to verify experimentally the advertised ROC of the curved output coupling mirror.
3. Could you use a convex ( $R < 0$ ) mirror in this setup? If so, are there any constraints on the radius of curvature that this mirror could have?

## 7.5 Polarization

1. What does it mean for the laser to be polarized? Check the polarization of the HeNe laser beam.
2. Why might the output of the laser be polarized? Are there any elements in the laser that could cause the output beam to be polarized?

## 7.6 Spectral Output

Make sure the cavity is short enough that a second mirror can be placed outside the cavity in order to reflect light into the monochromator.

1. Measure the spectrum of the laser light from 580 nm to 640 nm.
2. Measure the spectrum of the fluorescence directly from the tube over the same range. Why are the results different?

## 7.7 Mode Control and Beam Radius Measurements

In this section, your goal is to measure the beam radius inside the cavity and in at least two locations outside the cavity. You will compare your measured beam radius with what you expect given what you know about the ROC of the mirrors and the cavity length. You should use the curved mirror for this section of the experiment. Note: Outside the cavity you can easily place a camera in the beam in order to make a measurement of the beam size. You cannot do this inside the cavity (as it will stop the lasing action). You will find section 3.3 of Saleh and Teich indispensable for the required analysis in this section.

1. Place the wire (mounted on a translation stage on a carriage) into the cavity, somewhere between the output coupler mirror and the iris. Describe what you see as you translate the wire through the location of the beam. Explain why you see this behavior.
2. Align your setup such that you can see at least the TEM<sub>10</sub> and TEM<sub>20</sub> modes as you translate the wire across the beam (at different locations of the wire, of course). Once you think you have this alignment correct, please check with a TA.
3. Starting with the wire on one side of the cavity, translate the wire across the center of the cavity and record the micrometer position where each mode occurs. You should see the TEM<sub>20</sub> modes occur twice. Why is this? Which mode occurs when the wire is at the (transverse) center of the cavity?
4. Place the camera somewhere along the rail and take a picture of a TEM<sub>00</sub>, TEM<sub>10</sub> and TEM<sub>20</sub> mode at this location. Repeat for at least one other camera position.
5. From your data, determine the beam radius at each location (i.e., inside the cavity at the location of the wire, and at the positions of the camera). Hint: the pixel size of the camera is  $7.5 \pm 0.2 \mu\text{m}$ .
6. Use your previous measurements of the ROC of the mirrors, and the length of the cavity to determine the theoretical beam radius as a function of position.
7. Plot the theoretical beam radius as a function of position, and overlay the beam radius measurements that you obtained (both inside and outside the cavity). Also include an indication of the position of the cavity mirrors on your plot. Comment on the agreement (or disagreement) between your data and the theory.

## 7.8 Additional Questions

1. Show that the allowable laser frequencies are given by

$$f_m = m \frac{c}{2L} \quad (7.1)$$

where  $L$  is the length of the laser cavity, and  $m$  is the number of half-wavelengths that fit between the mirrors. Why can you only get red light out of this laser? In other words, Eq. ?? suggests that many frequencies should be possible, so why do you only see one color?

2. Do you see that the laser only lases for specific mirror separations, as predicted by Eq. ??? Why or why not? It might be useful to know that the FWHM of the gain medium in the HeNe laser is about 1 GHz.

### Final Question

What was the most challenging part of this lab? Is it what you thought it would be (i.e., what you wrote in your outline)? Comment (i.e., write more than yes or no).

## 7.9 Useful Readings

- Section 3.1 The Gaussian Beam
- Section 3.2 Transmission through Optical Components (Beam Shaping and Beam Focusing)
- Section 3.3 Hermite-Gaussian Beams
- Section 10.1 Planar-Mirror Resonators
- Section 10.2 Spherical-Mirror Resonators (including parts A and B)
- Section 13.3 Interactions of Photons with Atoms
- Section 15.1 Theory of Laser Oscillation
- Section 15.2 Characteristics of the Laser Output (especially part C on Spatial Distribution and Polarization and part D on mode selection)
- Appendices: A,C,D,E



## Appendix A

# CCD Camera Operation

For several experiments in this course you will use the CCD cameras to capture images or movies. Make sure to copy all files to your own directories or transfer them to a usb stick as the files will be periodically deleted.

**On** the computer, select ‘Run Command’ from the start menu, type **xawtv** and press ‘Enter’ (alternatively select *Applications* → *RunCommand* and type ‘xawtv’).

**Right** click on the image window to open the options menu

**Ensure** the “TV norm” is set to “NTSC”

**Adjust** the brightness and contrast by clicking in the respective scroll bars

- Left clicking will increase the brightness/contrast
- Right clicking will decrease the brightness/contrast

**Select** “Grab Image (JPEG)” to save an image. The image will be named *snap\_unknown\_timestamp\_nr.jpg* To change the base name (from *snap* to something else), initialize the program by typing **xawtv -o** followed by the new base.

For example, initializing with the string **xawtv -o HeNeModes** will cause images to be saved as *HeNeModes\_timestamp\_nr.jpg*

**Recording** a movie requires that “Capture” is set to “grabdisplay”

- Select “Record Movie (avi)”
- Set the movie driver to “Microsoft AVI (RIFF) format”
- Set the “movie/images filename” to *base.avi*
- Set the audio format to “No Sound”
- Set the sample rate to “44100”
- Set the video format to “MJPEG (AVI)”
- Set the frames per second to “12.0 fps”

**Make** sure to copy all files to your own directories or transfer them to a USB stick as the files will be deleted periodically from the machines.



## Appendix B

# Extended Fourier Analysis

Use the following example of Fourier spectra to compare the observed Fourier spectrum of the mesh to theory.

### Plane Wave

A plane wave that is propagating along the  $z$ -axis has the following Fourier transform:

$$E(x, y) = A_0 e^{ik_z z - \omega t} \equiv E_0[f(z, t)] \quad (\text{B.1})$$

Now we will drop  $f(z, t)$

$$F[E(x, y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_0 e^{-2\pi i(k_x x + k_y y)} dx dy = E_0 \delta(k_x) \delta(k_y) \quad (\text{B.2})$$

You can verify that the Fourier transform of a constant is a delta function by taking the inverse Fourier transform of a delta function:

$$F^{-1}[\delta(k_x)] = \int_{-\infty}^{\infty} \delta(k_x) e^{2\pi i k_x x} dx \quad (\text{B.3})$$

and since the delta function has the fundamental property that

$$\int_{-\infty}^{\infty} f(x) \delta(x - a) dx = f(a) \quad (\text{B.4})$$

it follows that

$$F^{-1}[\delta(k_x)] = e^{2\pi i(0)x} = 1 \quad (\text{B.5})$$

therefore

$$F[1] = F[F^{-1}[\delta(k_x)]] = \delta(k_x) \quad (\text{B.6})$$

Thus the transverse Fourier transform of a plane wave is a point on the focal plane at  $(k_x, k_y) = (0, 0)$ .

### Slit

A plane wave passing through a slit in one dimension of width  $w$  can be described by a rectangular function:

$$E(x, y) = E_0 \text{rect}\left(\frac{x}{w}\right) = \begin{cases} E_0 & |x| < w/2 \\ 0 & \text{else} \end{cases} \quad (\text{B.7})$$

Thus in the Fourier transform plane the spectrum should be:

$$F[E(x, y)] = E_0 \int_{-\infty}^{\infty} \int_{-w/2}^{w/2} e^{-2\pi i(k_x x - k_y y)} dx dy \quad (\text{B.8})$$

$$= E_0 \delta(k_y) \int_{-w/2}^{w/2} e^{-2\pi i k_x x} dx \quad (\text{B.9})$$

$$= E_0 \delta(k_y) \frac{1}{-2\pi i k_x} e^{-2\pi i k_x x} \Big|_{-w/2}^{w/2} \quad (\text{B.10})$$

After some manipulation, this can be shown to be

$$= E_0 \delta(k_y) \frac{\sin(\pi k_x w)}{\pi k_x} \quad (\text{B.11})$$

Using the definition  $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$ , the above equation further reduces to

$$F[E(x, y)] = E_0 w \delta(k_y) \text{sinc}(w k_x) \quad (\text{B.12})$$

### Grid

A grid is a periodic sequence of  $N$  slit functions with width  $w$  and separation  $s$  in the  $x$ -direction. Thus;

$$E(x, y) = E_0 \sum_{n=1}^N \text{rect}\left(\frac{x}{w} - \frac{n \cdot s}{w}\right) \quad (\text{B.13})$$

This can be expressed as a convolution of a series of delta functions with the rectangular function:

$$E(x, y) = E_0 \left[ \sum_{n=1}^N \delta(x - n \cdot s) \right] * \text{rect}\frac{x}{w} \quad (\text{B.14})$$

where the  $*$  represents a convolution. The Fourier transform of a convolution product<sup>1</sup> is

$$F[(E_1 * E_2)] = F[E_1] \cdot F[E_2] \quad (\text{B.15})$$

Therefore in the Fourier transform plane the spectrum should be

$$F[E(x, y)] = F\left[ \sum_{n=1}^N \delta(x - n \cdot s) \right] \cdot F\left[ E_0 \text{rect}\left(\frac{x}{w}\right) \right] \quad (\text{B.16})$$

---

<sup>1</sup>See the Appendix in Fundamentals of Physics



The second Fourier transform is the same as the slit from the last example. For the first term, the summation can be moved outside of the Fourier transform:

$$= \sum_{n=1}^N F[\delta(x - n \cdot s)] \cdot (E_0 w \delta(k_y) \text{sinc}(w k_x)) \quad (\text{B.17})$$

Using equation ??

$$= \sum_{n=1}^N e^{-2\pi i n s k_x} E_0 w \delta(k_y) \text{sinc}(w k_x) \quad (\text{B.18})$$

The infinite series of exponentials can be rewritten:

$$\sum_{n=1}^N e^{-2\pi i n s k_x} = e^{-\pi i s k_x (N+1)} \frac{\sin(\pi N s k_x)}{\sin(\pi s k_x)} \quad (\text{B.19})$$

so;

$$F[E(x, y)] = E_0 \delta(k_y) w \text{sinc}(w k_x) \cdot e^{-\pi i s k_x (N+1)} \frac{\sin(\pi N s k_x)}{\sin(\pi s k_x)} \quad (\text{B.20})$$

Thus the intensity is

$$I(k_x, k_y) = |E_0|^2 |\delta(k_y) w^2 \text{sinc}^2(w k_x) \frac{\sin^2(\pi N s k_x)}{\sin^2(\pi s k_x)}| \quad (\text{B.21})$$



## Appendix C

# The Monochromator and Photomultiplier

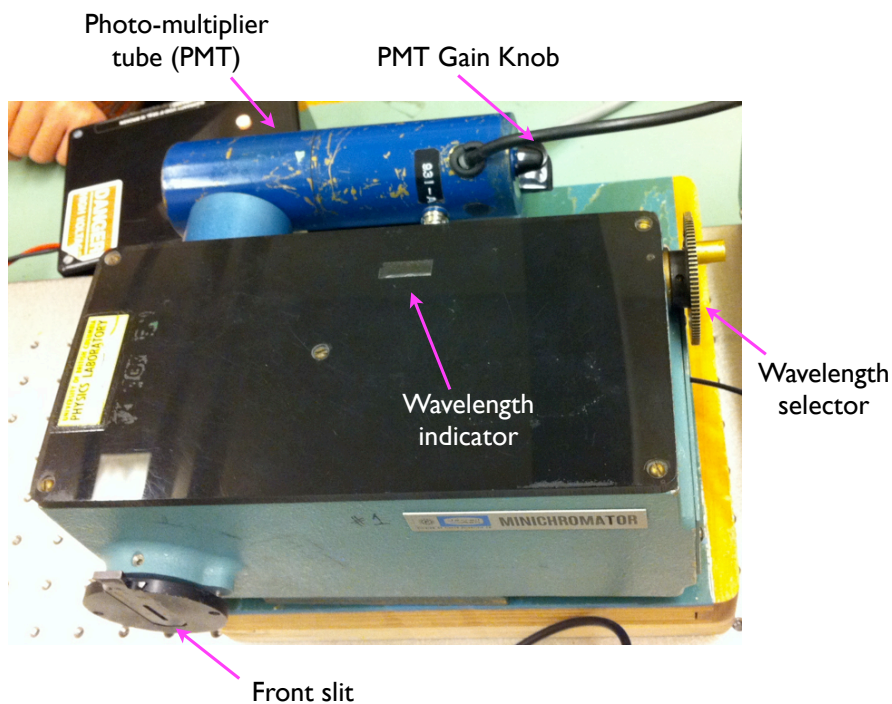


Figure C.1: The mini-monochromator available in the lab.

The easiest way to align the monochromator is to do this by eye (but don't do this with the laser - only with the pink light from the HeNe tube. So remove the OC from the HeNe laser). First, insure the PMT voltage source is OFF so you don't blind the PMT when you remove it from the monochromator. Next, direct your light source at the entrance slit and insure that it is propagating perpendicular to the slit plane. Then remove both the front and rear slit. Set

the monochromator wavelength to about 500 nm. Adjust the light source with a turning mirror so that you can see the light through the monochromator. Put in the front slit and reoptimize. Then put the back slit in and re-insert the PMT. Plug the PMT output into the scope and turn on the PMT power supply. Set it to about 1000V and dial the wavelength selector until you see a signal.

### Introduction

A monochromator is a device used to isolate a small wavelength interval of a spectrum. It consists of an entrance slit, a dispersive element (*e.g.* a grating), and an exit slit which allows only a selected portion of the spectrum to pass to a detector. The photon detector in this lab is a photomultiplier tube. The instrument can be used to study the wavelengths and intensities of spectral lines emitted by a source.

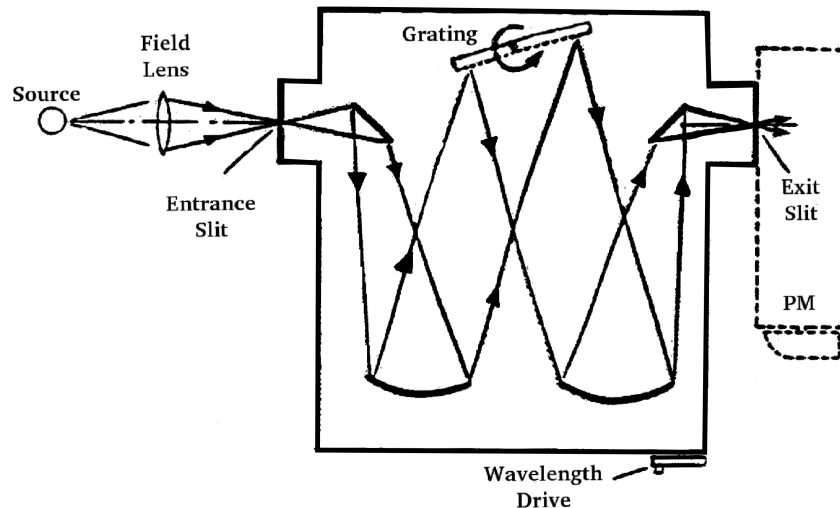


Figure C.2: The Czerny-Turner Monochromator (the field lens is optional).

### The Czerny-Turner Monochromator

Although the particular model of monochromator used here is small, the design and operation is identical to that of larger models. This particular arrangement of mirrors, slits, and grating is referred to as the Czerny-Turner arrangement. The dispersive element is a diffraction grating.

The light path through the instrument is shown in Fig. ???. The light from the source falls on the entrance slit. If necessary the light intensity can be increased with a field lens. A spherical mirror then collimates the beam of light and illuminates the grating. The grating disperses different wavelengths at different angles and one of these wavelengths is focussed onto the exit slit by the other spherical mirror. Since the slits are very narrow (typically 0.1mm or less), only a small range of wavelengths emerges from the exit slit. The instrument

width of this monochromator is approximately 1nm, but for more expensive instruments a width of  $10^{-2}$ nm is easily achievable.

### The RCA 931A Photomultiplier Tube

The photomultiplier tube is a vacuum tube device used in industry to measure light signals, especially those from transient light sources. (A transient light source is one which only emits for a short period of time, such as a spark). The active element of a PM is the photo-cathode, which liberates electrons when struck by light. These electrons form a current pulse, which is amplified by a series of dynodes. Because the photon energy depends on the colour of the light, the energy (and number) of liberated electrons will also. This changes the dynode avalanche amplification which results in a current pulse amplitude dependent on the colour of the light. The last dynode is called the anode and is connected by an anode resistor to ground (see Fig. ??). A voltage is therefore developed across the anode resistor which, in the ideal case, is proportional to the intensity of the incident light. The schematic construction and operation of the photomultiplier tube is shown in Fig. ??.

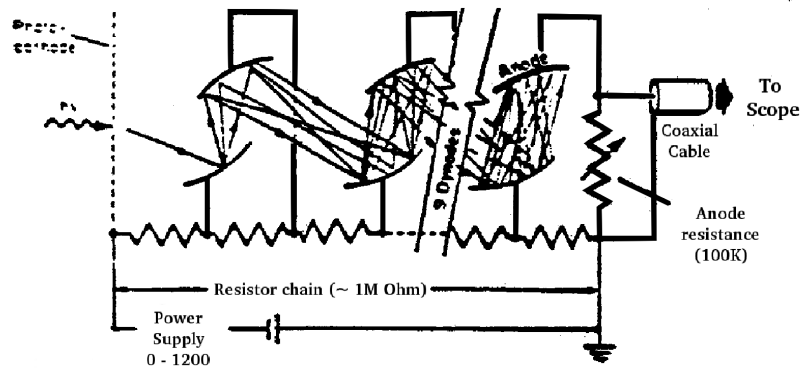


Figure C.3: Schematic of the RCA 931 Photomultiplier Tube.

The linearity of the PM response must be established before accurate intensity measurements can be made. This is done by plotting the amplitude of the PM signal as a function of the intensity of the incident light. Ideally the graph should be linear, but at large signals a deviation from linearity is usually observed.

### Monochromator alignment

The transmission through the monochromator is highly dependent on the angle of the light with respect to the monochromator. To line up the monochromator, first make sure the gain and the supply voltage on the PMT are set to ZERO and remove it from the monochromator. Set the monochromator to 500 nm and then take the slits out the front and back and look in through the back of the monochromator (i.e. put your eye where the PMT would normally be). Move the light source around until you can see light through the monochromator and

then put in the first slit and optimize the position of the lamp with this first slit in place. Put in the second slit and repeat the above step. When you are done, put the PMT back in place and turn the supply on and adjust the gain setting until you get a measurable signal (about 1V). The supply voltage to the PMT can be increased if more gain is needed, but do not exceed 1.2 kV. While you can use the voltmeter to measure the voltage output of the PMT, in most cases where the PMT output noise is high and you want a reliable measurement, you will need to use a fan to chop the light going to the monochromator, and you will use the oscilloscope to measure the PMT response to the dark and light periods.

### Spectral Response of the PM-Monochromator Combination

It was pointed out in Section ?? that the photomultiplier does not respond equally to light of different wavelengths; neither does the monochromator. The combination of the two is very wavelength sensitive. For many experiments involving comparison of intensity measurements at various portions of the spectrum, it is necessary to know the spectral response of the entire system.

To measure this response, light from a known continuum source is passed through the system and the PMT signal,  $S(\lambda)$ , is measured at many wavelengths over the complete spectrum. If the intensity of the source,  $I_w(\lambda, T)$  at a temperature of  $T$ , is known as a function of wavelength,  $\lambda$ , then the response of the system as a function of wavelength is given by,

$$R(\lambda) = \frac{S(\lambda)}{I_w(\lambda, T)} \quad (\text{C.1})$$

Once the response function is known, the intensity of any other source at wavelength  $\lambda$  may be determined by measuring  $S_2(\lambda)$  and dividing by  $R(\lambda)$ .

## Appendix D

# Modes of a Spherical Resonator

A spherical mirror resonator (like the one in the Cavity lab or the HeNe lab) support (i.e. resonate) with certain transverse mode patterns. These are the patterns corresponding to the exact solutions of the free-space paraxial wave equation. When the system has Cartesian symmetry, the solutions are the Hermite–Gaussian beams (or modes) composed of a 2D Hermite polynomial times a 2D Gaussian function. These are probably the most familiar to you, but there are others of importance (including the Laguerre gaussian modes and the Ince modes). And you may run across these other modes in the lab, so we provide a short discussion below as well as some pictures of the mode patterns for your reference.

### Hermite-Gaussian Modes

The Hermite-Gaussian modes (see Fig. ??) are particularly common, since many laser and/or resonator systems have Cartesian reflection symmetry in the plane perpendicular to the beam's propagation direction.

### Laguerre-Gaussian Modes

If the laser or resonator cavity is cylindrically symmetric, the natural modes are Laguerre-Gaussian modes (see Fig. ??). They are written in cylindrical coordinates using Laguerre polynomials

### Ince-Gaussian modes

the Ince–Gaussian beams (see Fig. ?? and Fig. ??) form the third complete family of exact and orthogonal solutions of the paraxial wave equation. They constitute the continuous transition modes between HGBs and LGBs, and are natural resonating modes in stable resonators. In particular, if the laser or resonator cavity has an elliptical symmetric, the natural modes are Ince–Gaussian modes. The transverse distribution of these fields is described by the Ince polynomials and has an inherent elliptical symmetry.

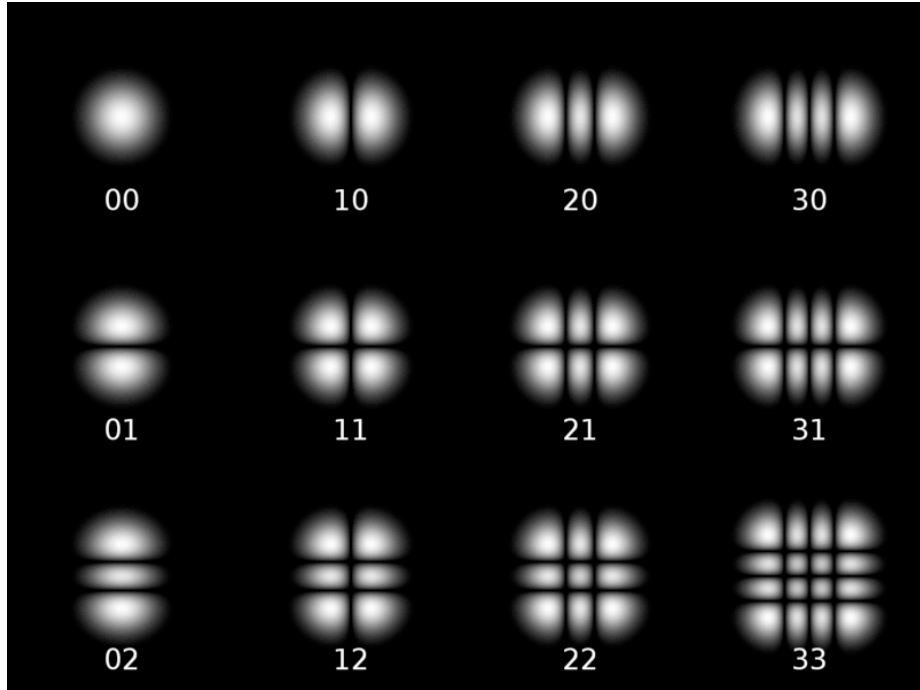


Figure D.1: Hermite-gaussian modes.

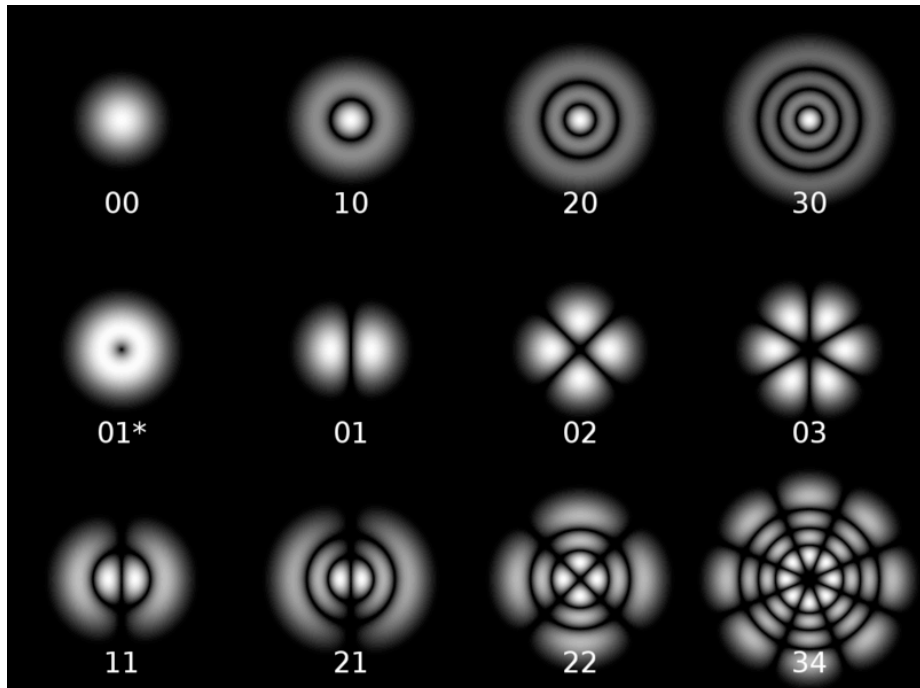


Figure D.2: Laguerre-gaussian modes.



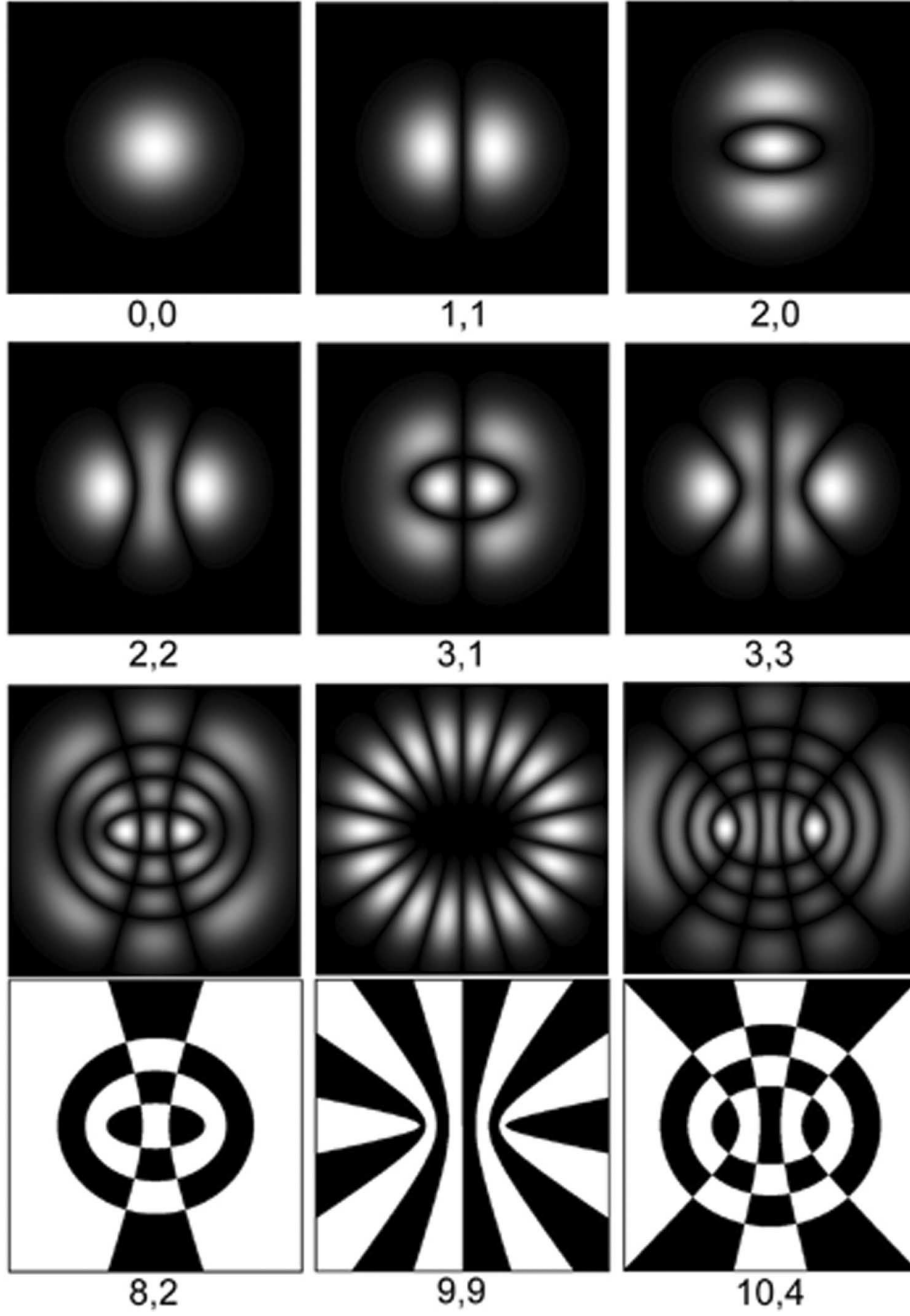


Figure D.3: Transverse field distributions of some even Ince-Gaussian beams. Plots in the bottom row correspond to the phase structures of the modes displayed in the row immediately above them. Figure from Opt. Lett. 29, 144 (2004).

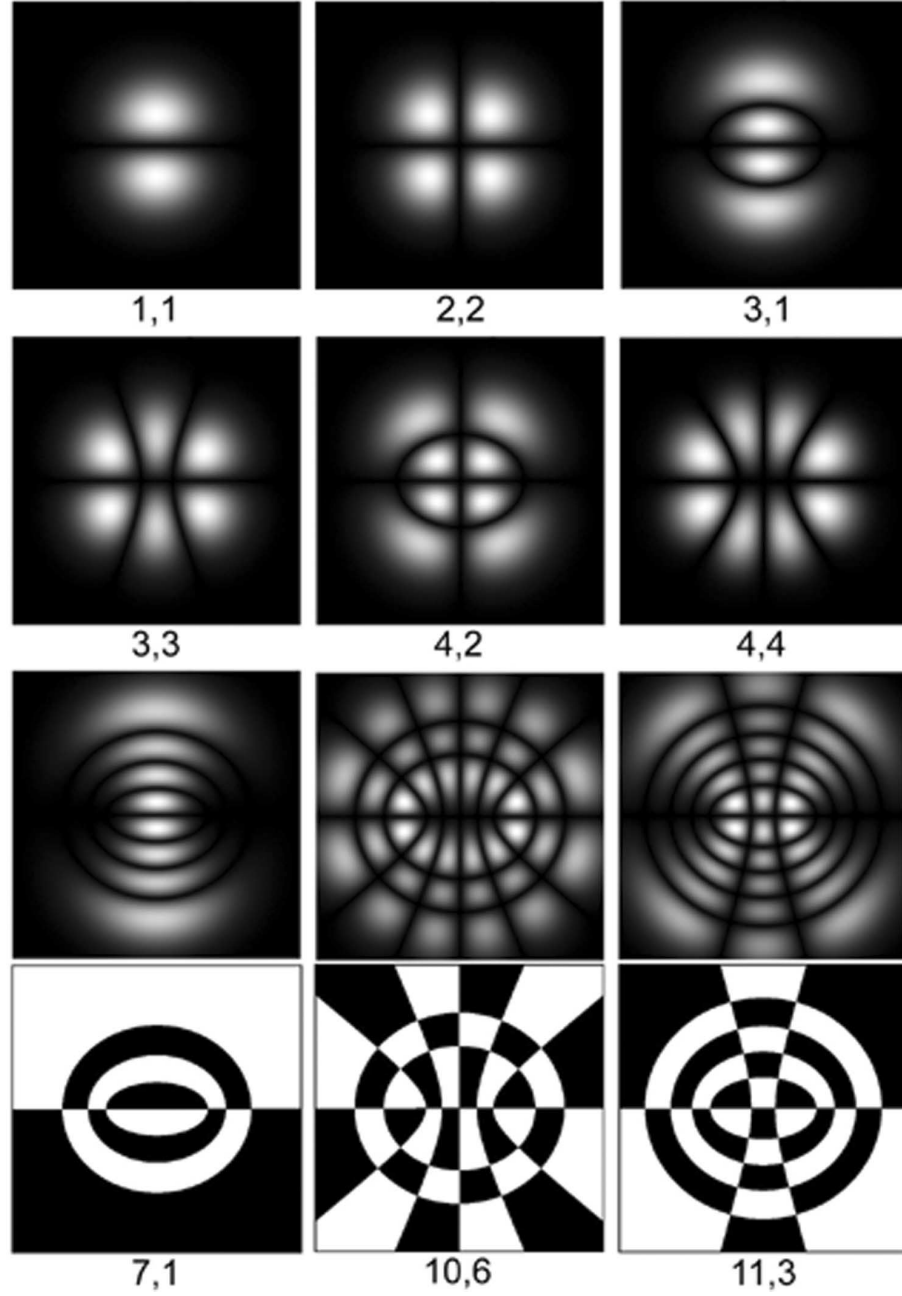


Figure D.4: Transverse field distributions of some odd Ince-Gaussian beams. Plots in the bottom row correspond to the phase structures of the modes displayed in the row immediately above them. Figure from Opt. Lett. 29, 144 (2004).

## Appendix E

# Resonator Theory

### Spherical-Mirror Resonators

An optical resonator composed of two planar mirrors ( $R_1 = R_2 = \infty$ ) is stable for any mirror separation so long as they have been perfectly aligned. The difficulty with this arrangement is that in practice planar mirrors are extremely sensitive to misalignment; they must be perfectly parallel to each other and perfectly normal to the incident light rays. This sensitivity can be reduced by replacing the planar mirrors with spherical ones. The trade off, however, is that spherical-mirror resonators are only stable for specific geometric configurations. These mirrors can be either concave ( $R < 0$ ) or convex ( $R > 0$ ).

Limiting yourself to ray optics, and specifically to the methods of paraxial matrix-optics, it is possible to determine that the region of stability for any spherical-mirror resonator is given by;

$$0 \leq \left(1 + \frac{d}{R_1}\right) \left(1 + \frac{d}{R_2}\right) \leq 1 \quad (\text{E.1})$$

where  $d$  is the optical cavity length, and  $R_1$  and  $R_2$  are the radii of curvature for the two mirrors. Typically, the two middle terms are written in terms of the ***g* parameters**

$$g_1 = 1 + \frac{d}{R_1} \quad \text{and} \quad g_2 = 1 + \frac{d}{R_2}$$

It is left as an exercise to demonstrate that this result is valid. You should record this derivation in your lab book. A good starting point for this analysis is located in your text book (Saleh and Teich, *Fundamentals of Photonics*).

The transmission function of the optical resonator in this lab (which is a Fabry-Perot interferometer) depends on the *quality* (or *Q*-factor) of the resonator (equivalently the *finesse*) and the spectrum of the laser light. For a laser input with an infinitely narrow optical spectrum, the cavity transmission is

$$T = \frac{T_{\max}}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2(\Delta\phi_{\text{rt}}/2)} \quad (\text{E.2})$$

where  $T_{\max}$  is the maximum transmission (depending on the mirror reflectivity),  $F$  is the cavity *finesse* and  $\Delta\phi_{\text{rt}}$  is the round trip optical phase. The *finesse* is defined by

$$F = \frac{\pi\sqrt{r}}{1 - r} \quad (\text{E.3})$$

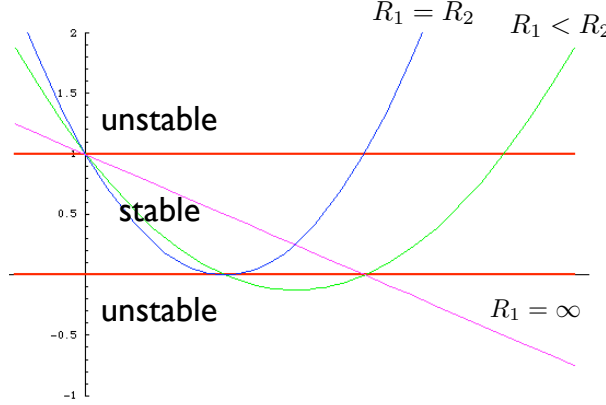


Figure E.1: Plot of the middle term in expression (??) as a function of the mirror separation,  $d$ , for various mirror combinations (i.e. values of  $R_1$  and  $R_2$ ). The cavity is stable at all locations,  $d$ , for which the value of the term is between 0 and 1 (denoted by red lines).

where the amplitude of the wave is reduced by a factor  $r$  on each round trip. Given intensity reflection coefficients  $R_1$  and  $R_2$ , we have that  $r = \sqrt{R_1 R_2}$ . For a plane wave inside a cavity of length  $L$  made of planar mirrors,  $\Delta\phi_{rt} = 2kL$ , where  $k = n2\pi/\lambda$  and  $n$  is the index of refraction of the material inside the cavity. The cavity transmission is maximum when  $\Delta\phi_{rt}/2 = q\pi$  where  $q$  is an integer or equivalently when  $2kL = 2\pi q$ .

**Okay, here's the main point:** the resonance condition is then  $L/\lambda = q/2$  (where  $q = 1, 2, 3, \dots$ ). That is, the cavity length must be an *integer multiple of half the wavelength of the input light*! That means that monitoring the resonance of optical cavities is great way to detect small changes (on the order of  $\lambda$ ) in the cavity length. This is the principal of operation for LIGO, the cosmic gravitational wave detectors run by MIT and Caltech. **Notice:** since we can *either* vary the input **laser frequency** (i.e. wavelength) **OR** the **cavity length** to move the laser and the cavity into resonance, we get *two conditions on the positions of the resonances*. For a **fixed cavity length**, the resonance condition on the wavelength or frequency of the input beam is

$$\lambda_q = \frac{2L}{q}; \quad \nu_q = \frac{cq}{2L} = q \nu_{\text{FSR}} \quad (\text{E.4})$$

where the so-called “free-spectral range” is  $\nu_{\text{FSR}} = c/2L$  and the speed of light in the cavity is  $c = c_0/n$  where  $c_0$  is the speed of light in a vacuum. The time it takes a photon to travel from M1 to M2 and back to M1 (the round trip time) is simply  $\tau_{rt} = 1/\nu_{\text{FSR}}$ . From this, we see that the cavity transmission is periodic in the input laser frequency with period  $\nu_{\text{FSR}}$ . On the other hand, for a **fixed laser frequency**, the resonance condition on the length of the cavity is that it must be an integer number of half wavelengths

$$L_q = q \frac{\lambda}{2}. \quad (\text{E.5})$$

This implies that for a fixed input frequency, the cavity transmission is periodic

in the length  $L$  of the cavity with period  $\lambda/2$ . Figure ?? shows a schematic of these cavity resonances and how they change when the cavity length is changed, and the shape of the cavity transmission is shown in Fig. ??.

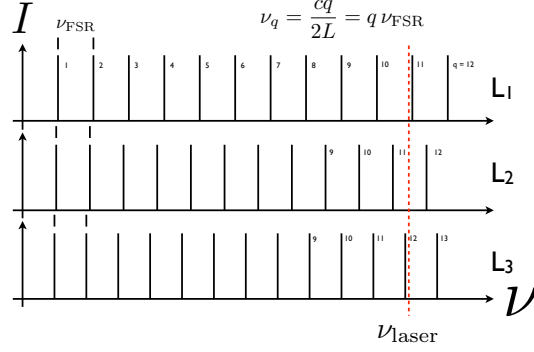


Figure E.2: This figure shows the set resonance frequencies  $\nu_q = q \nu_{\text{FSR}}$  for the cavity (with the resonances labeled here by the  $q$  value) for three slightly different lengths ( $L_1 < L_2 < L_3$ ). As the length of the cavity **increases**, the free spectral range ( $\nu_{\text{FSR}}$ ) **decreases** and the 11<sup>th</sup> and 12<sup>th</sup> resonant mode are at first above the laser frequency ( $\nu_{\text{laser}}$ ) for a cavity length of  $L_1$  and then they both move below it for a cavity length of  $L_3$ . Ramping the cavity length from  $L_1$  to  $L_3$  would then produce two identical transmission peaks. Note: this figure is only a schematic since the typical value of  $q$  is on the order of  $10^5$  to  $10^6$  and not 11 or 12. The value of  $q$  is simply the number of wavelengths that fit inside  $L$  - that is  $q = L/\lambda$ .

The minimum cavity transmission is achieved when  $\sin^2 \Delta\phi_{\text{rt}}/2 = 1$  and is

$$T_{\min} = \frac{T_{\max}}{1 + \left(\frac{2F}{\pi}\right)^2}. \quad (\text{E.6})$$

The minimum intensity only goes to zero in the limit of large finesse - that is when the *mirror reflectivity becomes nearly perfect* ( $r \rightarrow 1$ ). As a way to characterize the width of the resonances, we can find the full width at half maximum of the transmission peaks. The points at which the transmission falls to  $T_{\max}/2$  (i.e. when  $\sin^2(\Delta\phi_{\text{rt}}/2) = (\frac{\pi}{2F})^2$ ) are given by  $\Delta\phi_{\text{rt}} = 2 \sin^{-1}(\pi/2F)$ . The width of a resonance is really only a sensible concept in the limit of large finesse, when the resonances are well resolved. In this limit, we can write the half-maximum intensity phases as  $\delta_{\text{HM}} \simeq \pm(\pi/F)$ . And thus the full width of the resonances at half maximum is  $\delta_{\text{FWHM}} = 2\pi/F$  or equivalently  $L/\lambda = 1/(2F)$ . Again, since we can either vary the input **laser frequency** OR the **cavity length** to move the laser and cavity through resonance, we get the following conditions on the width of the resonances:

$$\nu_{\text{FWHM}} = \frac{\nu_{\text{FSR}}}{F} \quad (\text{E.7})$$

$$L_{\text{FWHM}} = \frac{\lambda}{2F} \quad (\text{E.8})$$

$$\lambda_{\text{FWHM}} = \frac{1}{2LF} \quad (\text{E.9})$$

Fig. ?? shows the transmission (or intensity inside the cavity) as a function of the cavity length given a *fixed laser frequency* and as a function of the input frequency  $\nu$  given a *fixed cavity length*  $L$ . From this, it is clear that the cavity finesse can be obtained experimentally by taking the ratio of the cavity periodicity and dividing this by the width of the transmission peaks.

$$F = \frac{\nu_{\text{FSR}}}{\nu_{\text{FWHM}}} = \frac{\frac{\lambda}{2}}{L_{\text{FWHM}}} \quad (\text{E.10})$$

Alternatively, if the mirror reflectivity (thus finesse) and cavity length  $L$  are known, the frequency or length resolving power of the cavity can be computed. Fig. ?? shows the transmission of the cavity at different values of the finesse.

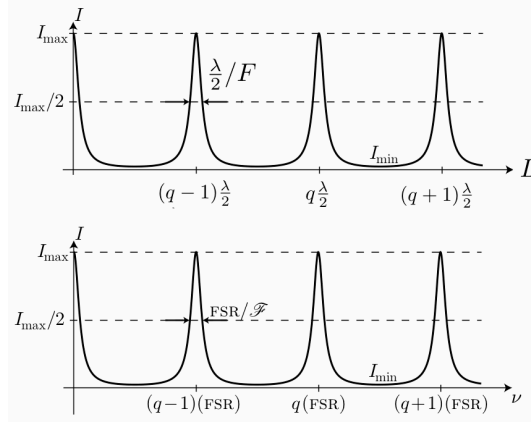


Figure E.3: Transmission of cavity.

As the finesse is increased, the resonances become more and more sharp and the transmitted light off of resonance becomes smaller and smaller.

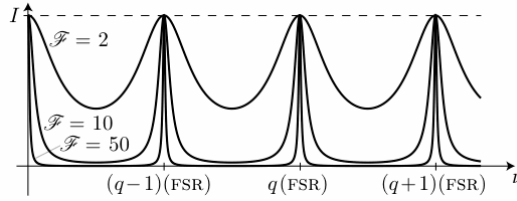


Figure E.4: Transmission of cavity for various values of the finesse.

### Cavity resonances for different spatial modes

So far, we have assumed that we have a plane wave inside a cavity of length  $L$ . In this case, the round trip phase of the wave is  $\Delta\phi_{\text{rt}} = 2kL$ , and the resonance condition for the cavity is given by Eqn. ?. However, the optical wave inside the cavity is actually a Gaussian beam and it may have transverse

mode structure (i.e. curves or lines along which the electric field and intensity vanish). Note: the round trip phase is slightly *different* for each mode! Figure ?? shows example plots of the intensity pattern of a  $\text{TEM}_{l,m}$  Gaussian beam with different transverse mode numbers  $l$  (the number of nodes along the  $x$  axis) and  $m$  (the number of nodes along the  $y$  axis). The beam is assumed to be propagating along  $z$ .

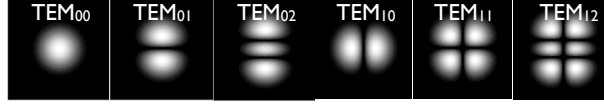


Figure E.5: Transverse intensity pattern of a  $\text{TEM}_{l,m}$  Gaussian beam. The mode is specified by the numbers  $l$  and  $m$  corresponding to the number of nodes along  $x$  and  $y$  respectively.

For a  $\text{TEM}_{l,m}$  mode, the resonant condition (Eqn. ??) is modified by the additional phase associated with the transverse mode

$$\nu_q = q\nu_{\text{FSR}} + (l + m + 1) \frac{\Delta\xi}{\pi} \nu_{\text{FSR}} \quad (\text{E.11})$$

where  $\Delta\xi$  is the phase retardation of this Gaussian mode relative to a plane wave (otherwise known as the accumulated Gouy phase). Think of it as if the different modes traverse slightly diagonal trajectories in the cavity and they therefore experience slightly different cavity lengths. For a rigorous explanation, see the discussion surrounding Eqn. 10.2-33 in your textbook. Note that for  $l = m = 0$  we recover Eqn. ??, the original result for a plane wave. **The main point** here is that different transverse modes are resonant with the cavity at slightly different frequencies  $\nu_q$ . Equivalently, a laser at fixed frequency will resonantly excite different modes of the cavity at slightly different cavity lengths (which control  $\nu_{\text{FSR}}$ ). This is why in Fig. ?? we see a series of distinct transmission peaks at slightly different cavity lengths occurring periodically as the cavity length is increased by  $\lambda/2$  (or one free spectral range). Each distinct peak corresponds to a different  $\text{TEM}_{l,m}$  mode and it is excited when the cavity length is just right so that the laser frequency is equal to the cavity resonance frequency for that mode given by Eqn. ??.

### Photon survival time and $Q$ -factor

A photon in the cavity completes one round trip every  $\tau_{\text{rt}} = 1/\nu_{\text{FSR}}$  seconds. Over this round trip, it has a probability  $P_s = R_1 R_2$  of surviving the trip (i.e. not being lost from the cavity). Here  $R_1$  and  $R_2$  are the intensity reflection coefficients. Therefore the lifetime of a photon inside the cavity is

$$\tau_p = \frac{\tau_{\text{rt}}}{1 - P_s} = \frac{1}{\nu_{\text{FSR}}(1 - P_s)} \quad (\text{E.12})$$

The finesse also depends on the mirror reflectivity and can be written as

$$F = \frac{\pi P_s^{1/4}}{1 - \sqrt{P_s}}. \quad (\text{E.13})$$

For large finesse (large survival probabilities), we can approximate  $P_s^{1/4} \simeq 1$  and  $(1 - P_s) \simeq 2(1 - \sqrt{P_s})$  which allows us to rewrite the photon lifetime as

$$\tau_p = \frac{1}{2\nu_{\text{FSR}}(1 - \sqrt{P_s})} = \frac{F}{2\pi\nu_{\text{FSR}}} = \frac{1}{2\pi\nu_{\text{FWHM}}} \quad (\text{E.14})$$

and we get an “uncertainty relation” (analogous to the time/energy uncertainty principle in quantum mechanics) of

$$\tau_p \nu_{\text{FWHM}} = \frac{1}{2\pi} \quad (\text{E.15})$$

The resonator *quality* or  $Q$ -factor is  $2\pi$  times the ratio of the total energy stored in the cavity divided by the energy lost in a single cycle. We can write this as

$$Q = 2\pi\nu_q\tau_p = \frac{\nu_q}{\nu_{\text{FWHM}}} = qF. \quad (\text{E.16})$$



## Appendix F

# Images of Components for Cavity Lab

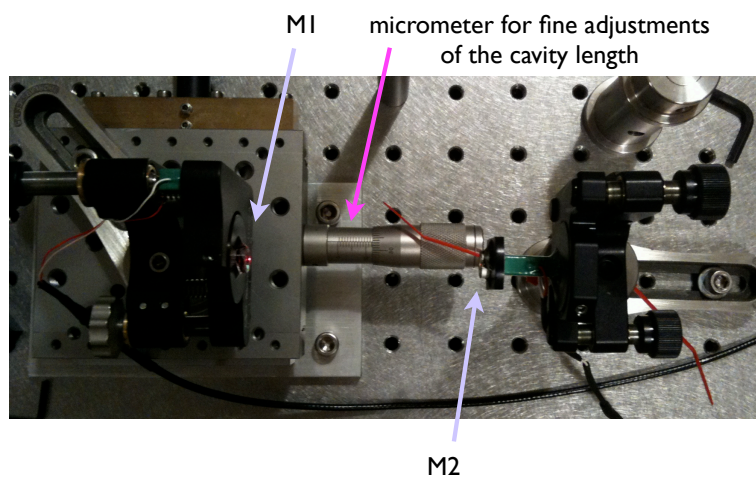


Figure F.1: Image of the cavity showing M1 and M2.

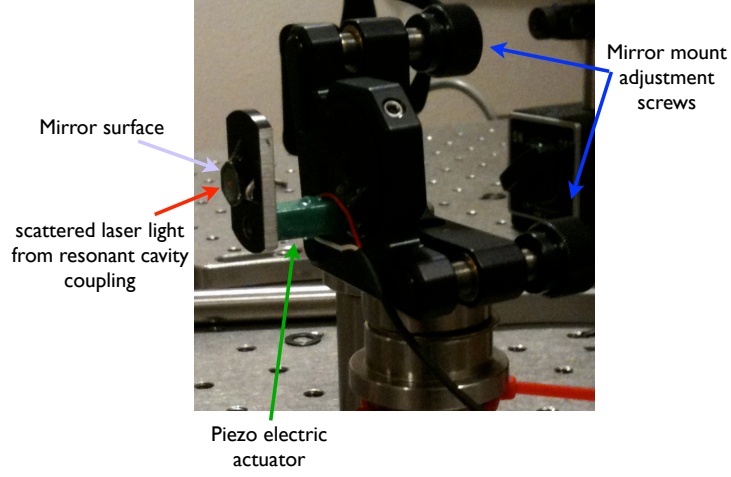


Figure F.2: Close-up image of the mirror M2 showing the piezo actuator. When the cavity is resonant with the input laser light, the optical power inside the cavity is very large and scattered light from the mirrors becomes clearly visible.

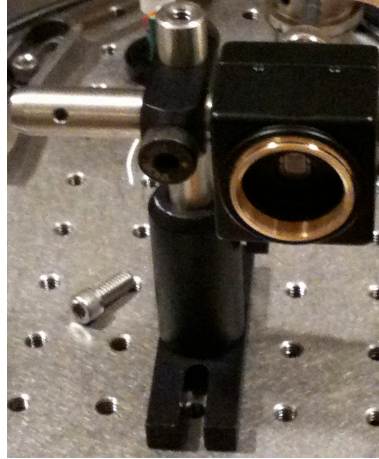


Figure F.3: Image of the CCD camera. The active area of the sensor is the small rectangle (approximately 2x3 mm) centered in the region bordered by the threaded brass ring.

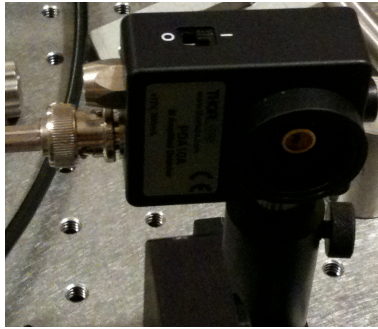


Figure F.4: Close-up image of the photodiode. The active area of the sensor is a tiny ( $\approx 1 \text{ mm}^2$ ) square centered in the area bordered by the gold ring.

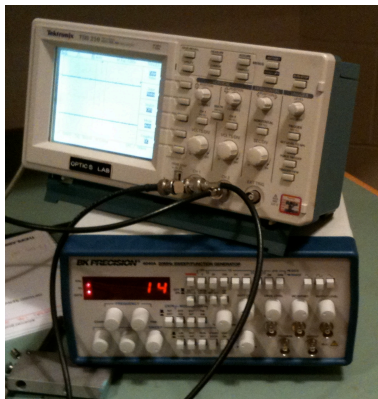


Figure F.5: Oscilloscope and function generator. The output of the function generator should be sent to both the scope and the high voltage amplifier. The photodiode signal should also be displayed by the scope. An example trace is shown in fig. ??



Figure F.6: High voltage (HV) amplifier (THORLABS model MDT694A). The display shows the output voltage. A voltage applied to the “EXT INPUT” on the front is amplified and that voltage is added to that set by the manual “OUTPUT ADJ” knob. The input multiplier changes depending on the voltage limit indicated by the green LED.

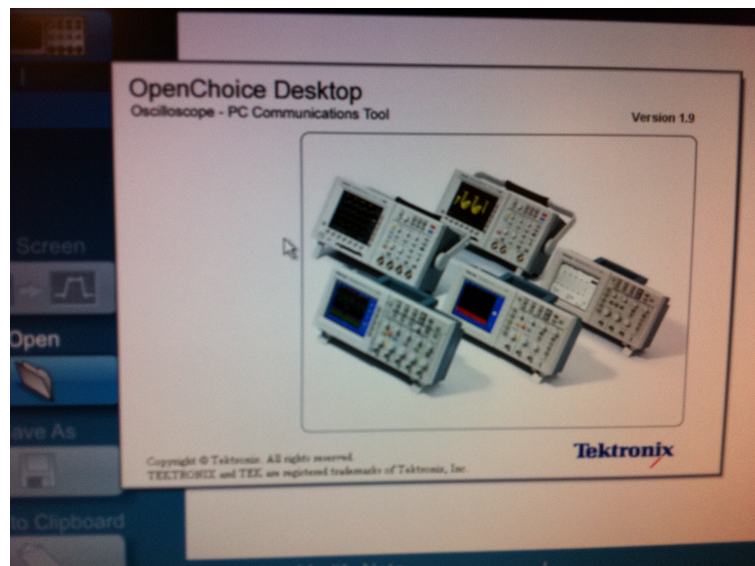


Figure F.7: Welcome screen for "Open Choice Desktop: oscilloscope - PC communications Tool"

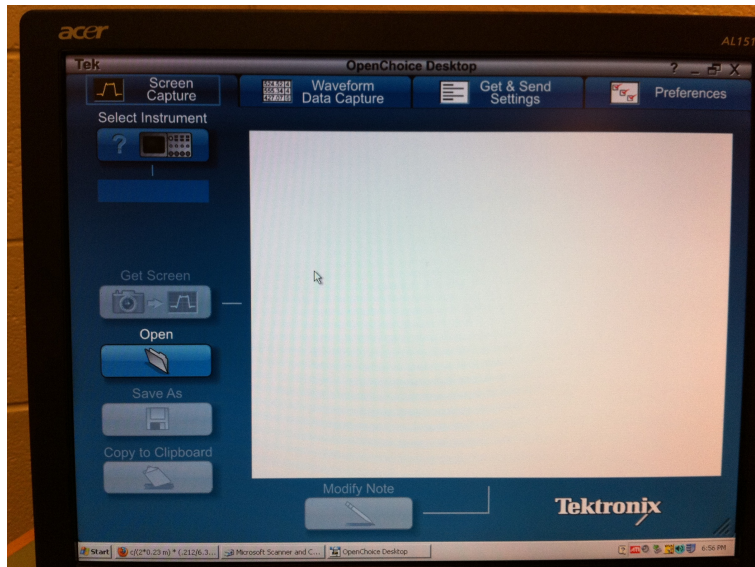


Figure F.8: **IMPORTANT NOTE:** Here the oscilloscope is not visible when Open Choice Desktop is launched. Specifically, the box under the "Select Instrument" icon is empty. If this box remains empty upon power cycling the scope, then you need to leave the oscilloscope on and reboot the computer.

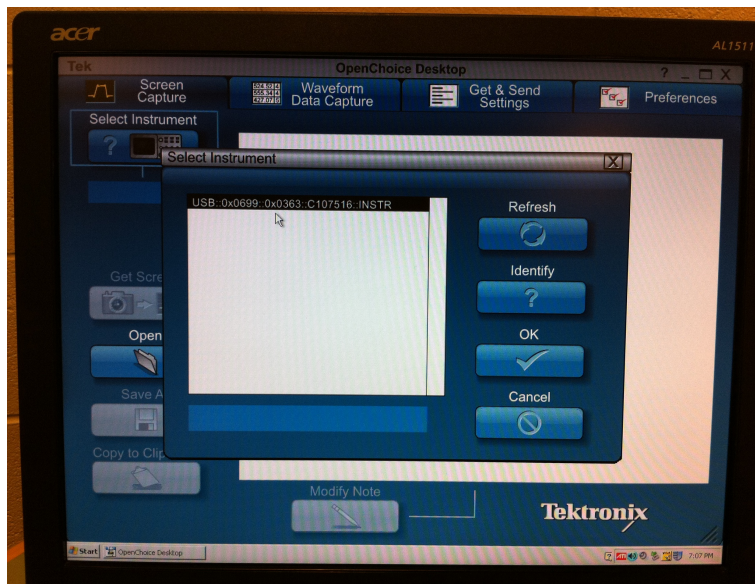


Figure F.9: Here, the oscilloscope became visible and was selected. After this



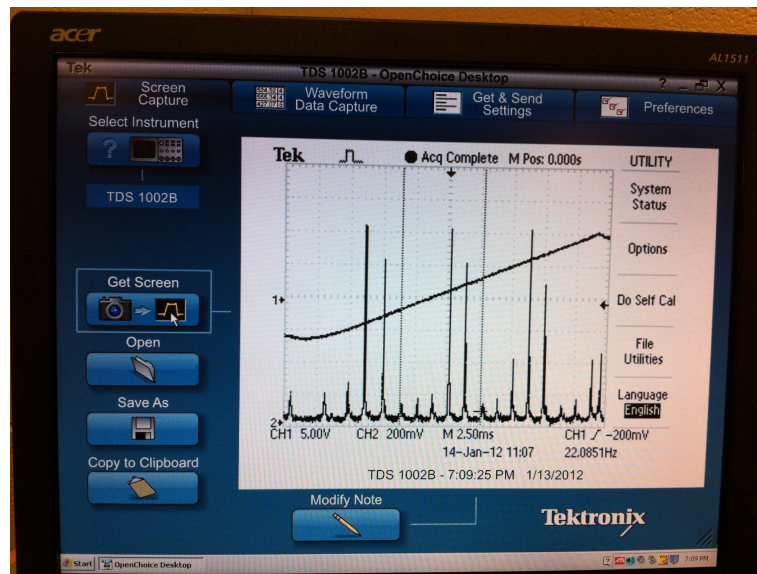


Figure F.10: Here, an oscilloscope screen shot was taken. Alternatively, the data can be captured and saved using "Waveform Data Capture"

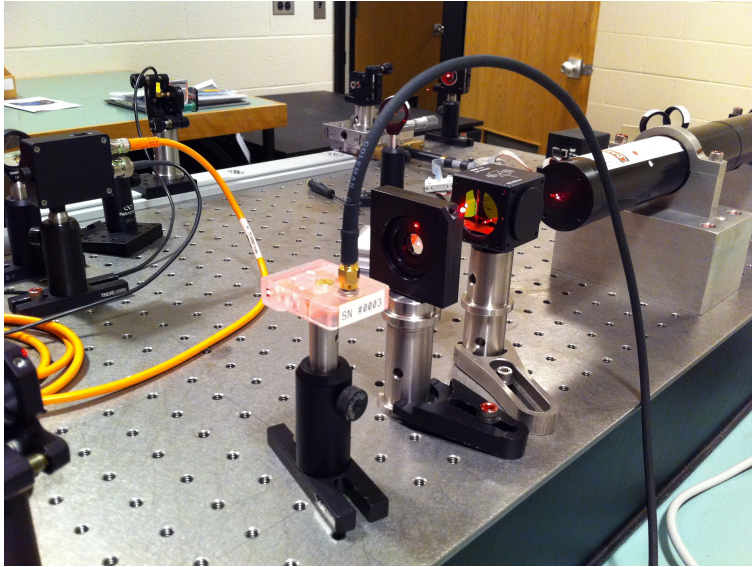


Figure F.11: Cavity lab setup: In the foreground, you can see (starting from the far right) the HeNe laser tube, then a polarization beam splitting cube, then a quarter-wave plate followed by the translucent electro-optic modulator (EOM) crystal holder (there is a black SMA cable going up from the top surface). At the bottom left of the image is a turning mirror that sends the laser beam into the background. The SMA cable partially obstructs the view of a coupling lens and the second turning mirror in the background. The cavity mirrors are visible in the background. The final mirror is glued to a long piezo-electric element. Its green color and the thin red and black wires going to it are visible in the image. The orange/yellow cable is the power to a Thorlabs photodetector.

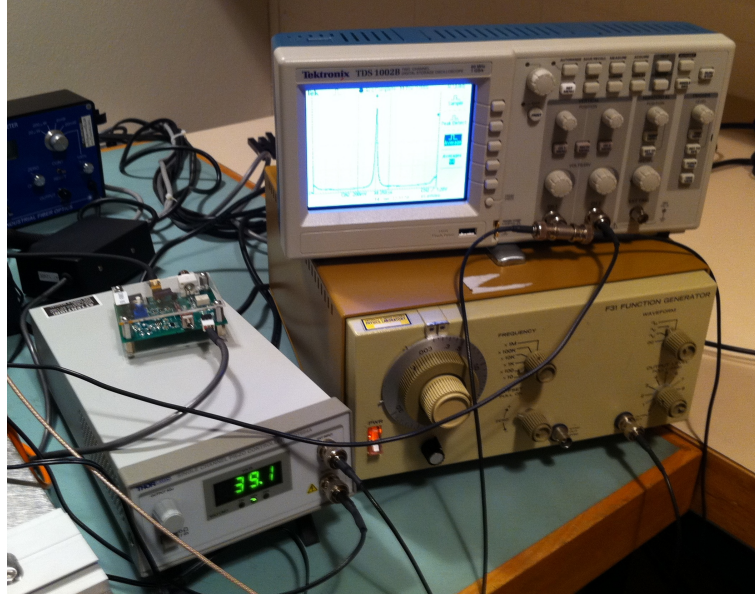


Figure F.12: Cavity Electronics showing the high voltage driver for the piezo-electric element, the oscilloscope, a function generator, and the electro-optic modulator driver.

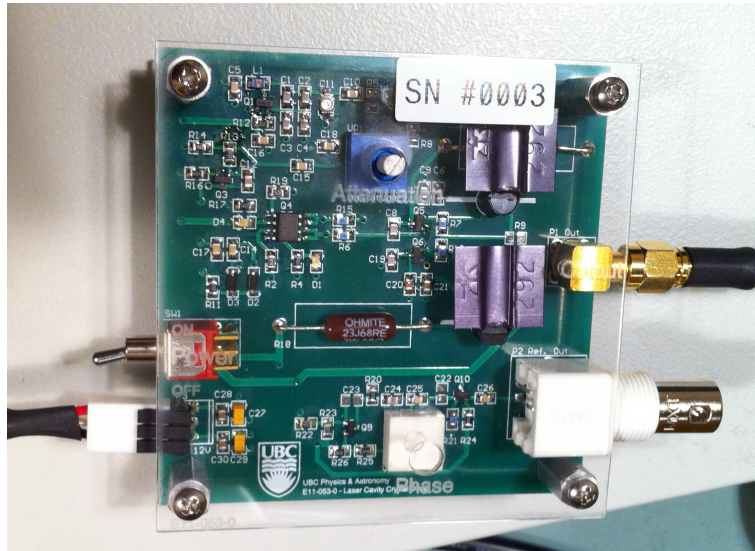


Figure F.13: Close up picture of the electro-optic modulator driver circuit. This device produces a high voltage ( $\sim 500$  V), radio-frequency ( $\sim 20$  MHz) sinusoid which drives the electro-optic modulator crystal. The black SMA cable leaving on the right of the image carries the drive signal to the crystal. The red switch enables the device and the black cable on the left of the image is the power supply connection. A bright green led lights when the device is powered on.



## Appendix G

# Using the Oscilloscope

This section gives a few examples of scope usage.

### Setting the scope to trigger on the PZT voltage ramp

1. Verify that the output of the function generator is going to channel 1 and the photodiode (hitherto referred to as "PD") output is going to channel 2
2. press the "TRIGGER MENU" button
3. toggle "source" button to channel 1 (the voltage ramp channel)
4. toggle mode to "Normal" (there is "auto", "normal", and "single")
5. move the TRIGGER LEVEL knob up until the scope is properly triggered on the voltage ramp. Adjust the "HORIZONTAL POSITION" knob so that you can see at least one half period of the voltage triangle wave
6. The trace from channel 1 and channel 2 should look like those in Fig. ??

### Setting the scope to trigger on the photodiode signal - the transmission peaks

1. press the "TRIGGER MENU" button
2. toggle "source" to channel 2 (the PD channel)
3. toggle mode to "Normal" (there is "auto", "normal", and "single")
4. move the TRIGGER LEVEL knob up until the main peak appears consistently appears centered on the screen. You may need to move the "HORIZONTAL POSITION" knob to get the peak to be centered when you zoom in on the peak by dialing the "HORIZONTAL SEC/DIV" knob

Sometimes it will be useful to average a few traces. This can be done with the following procedure.

1. press the "ACQUIRE" button at the top in the "MENUS" section

2. select "Average" (there is "Sample", "Peak detect", and "Average")
3. toggle the "Averages" button to something not too large - like 8 or 16

## Appendix H

# Calibration Of The Interferometer

The overall plan is to move the stage a certain distance and count the number of fringes that pass. You should therefore take the data, plot it, and count the fringes on the plot. The mirror moves a distance of  $\lambda/2$  as one fringe passes a reference point on the CCD Camera. Thus, if  $N$  fringes pass through as the mirror moves a distance  $d$ , we have,

$$2d = N\lambda \quad (\text{H.1})$$

If the servo controller readings change by  $\Delta D$  from  $D_1$  to  $D_2$ , then the mirror travel is linearly related to the stage displacement through the calibration constant  $K$ :

$$d = K\Delta D = K | D_2 - D_1 | \quad (\text{H.2})$$

To analyze the fringe movement (i.e. to count the fringes), you will use **MATLAB** to control the translation stage (i.e. mirror) position AND to detect the variation of the intensity on a given pixel in the fringe pattern as the fringe pattern moves. Several helpful **MATLAB** files are located in the desktop folder "Michelson files". At this point you should NOT be running the **WinTV2000** program as it will not allow **MATLAB** to access the video card. By reading the comments of the M-files you will be able to tell what each of their functions are.

In order to take data for this section, you will be using the **MATLAB** script called "michelson\_timestream.m". You will need to quit the APT user program when you use the "michelson\_timestream" file since **MATLAB** will open APT user again with the correct parameters. When this script is executed, it will prompt you to click on the image the location where the fringe pattern is the most visible. The script will then move the stage back by a distance (**span**) specified in mm, then it will move the stage forward by twice that distance ( $2 \times \text{span}$ ) at a slow velocity (**speed**) specified in mm/s, and then it will move the stage back to the starting point. To use this script, your code will look like this

```
>> data = michelson_timestream(span, speed);
```

where **span** and **speed** are to be chosen by you, and the data will be returned into the array **data**. As an example, consider this code

```
>> data = michelson_timestream(0.01, 0.001);
```

This will move the stage back by  $10\ \mu\text{m}$  and then forward by  $20\ \mu\text{m}$  at a speed of  $1\ \mu\text{m/s}$  (which will take 20 seconds), and then back  $10\ \mu\text{m}$  to the starting point. The data will be returned into the array **data** which will have the intensity on the pixel you selected for each frame of the video. Since the video frame rate is 30 frames per second, this means the stage will have moved  $\frac{1\ \mu\text{m/s}}{30\ \text{frames/s}} = \frac{33\ \text{nm}}{\text{frame}}$ .

For a given amount of distance moved by the stage, determine the number of fringes that passed through the reference point (region of interest, ROI) and then calculate the calibration constant  $K$ . Repeat this measurement as many times as necessary to obtain consistent results for  $K$  and estimate its accuracy. **Note:** because the backlash in the screw makes the **span** distance somewhat inaccurate you should be using the **speed** to determine the amount the stage is moving in some interval of time instead of assuming the stage is moving the entire **span** from start to finish.

## Appendix I

# Fourier Transform Spectroscopy

So far you have investigated the behavior of the interference fringe pattern as a function of the optical path difference for different optical sources. In general the fringe pattern intensity versus the optical path difference (or equivalently versus the time delay between the two interfering beams) is related to the *power spectrum* of the light entering the amplitude division interferometer by a Fourier transform. Therefore one can easily measure the optical power spectrum using the measurement of the intensity variation as a function of stage position.

Let's see how this works <sup>1</sup>. The electric field incident on the camera is the sum of the electric fields  $\mathbf{E}_1$  and  $\mathbf{E}_2$  arriving from M1 and M2 respectively after having passed through the beam splitter. Assuming for the moment that the input source is a purely *monochromatic* wave, we can characterize these electric fields with a vector amplitude (encoding the strength and polarization of the wave) and a spatial and time dependent phase:

$$\mathbf{E}_1 = \mathbf{A}_1 e^{i(\mathbf{k}_1 \cdot \mathbf{r} - \omega t + \phi_1)} \quad (\text{I.1})$$

$$\mathbf{E}_2 = \mathbf{A}_2 e^{i(\mathbf{k}_2 \cdot \mathbf{r} - \omega t + \phi_2)}. \quad (\text{I.2})$$

The phase includes terms ( $\phi_1 = |\mathbf{k}_1| l_1$  and  $\phi_2 = |\mathbf{k}_2| l_2$ ) which depend on the optical path lengths from the beamsplitter along path  $l_1$  (encountering M1) and path  $l_2$  (encountering M2). The intensity on the camera is simply the square of the total electric field:

$$I = |\mathbf{E}|^2 = \mathbf{E} \cdot \mathbf{E}^* = (\mathbf{E}_1 + \mathbf{E}_2) \cdot (\mathbf{E}_1^* + \mathbf{E}_2^*) \quad (\text{I.3})$$

$$= |\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + 2\mathbf{A}_1 \cdot \mathbf{A}_2 \cos \theta \quad (\text{I.4})$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \theta \quad (\text{I.5})$$

where we have assumed in the last step that the waves have the same polarization and that  $\theta = (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} + \phi_1 - \phi_2$ . You might be wondering about a white light source which is probably completely unpolarized. Is this assumption still valid in that case? This assumption is justified since the two fields  $\mathbf{E}_1$  and  $\mathbf{E}_2$  are copies of the incident field generated by the beam splitter. As long as the

---

<sup>1</sup>This derivation follows that in "Introduction to Modern Optics" by Grant R. Fowles

beam splitter and the optics which follow preserve the polarization of the light, these two fields will arrive at the detector with exactly the same polarization (whatever it was at the input of the interferometer).

If the interferometer is well aligned so that the waves are co-linear ( $\mathbf{k}_1 = \mathbf{k}_2$ ) and the 50/50 beamsplitter generates two waves of the same intensity ( $I_1 = I_2 = I/2$ ), then the interference pattern is simply

$$I(x) = I(1 + \cos kx) \quad (\text{I.6})$$

where  $x = l_1 - l_2$  is the path length difference and  $k = |\mathbf{k}_1| = 2\pi/\lambda$  is the magnitude of the wavevector.

Okay, now in general, the light illuminating the interferometer will be composed of *many* different frequencies. So, let's model the power spectrum of the incident light as a continuous distribution,  $G(\omega)$ . Then the total optical power emitted into the interferometer in the frequency range from  $\omega_0$  to  $\omega_0 + d\omega$  (where  $d\omega$  is an infinitesimally small frequency band) is simply  $G(\omega_0)d\omega$ . For convenience, we can instead use the distribution over wavelength  $G(k)$  (where  $\omega = ck$ ). Then the optical power,  $P$ , hitting the detector from illuminating the interferometer with this polychromatic source will be the sum of the interference patterns from each monochromatic wave composing  $G(k)$ . Assuming the interferometer is ideal and has no losses, we have then:

$$P(x) = \int_0^\infty G(k) (1 + \cos kx) dk \quad (\text{I.7})$$

$$= \int_0^\infty G(k) dk + \int_0^\infty G(k) \frac{e^{ikx} + e^{-ikx}}{2} dk \quad (\text{I.8})$$

$$= \frac{1}{2}P(0) + \frac{1}{2} \int_{-\infty}^\infty G(k) e^{ikx} dk \quad (\text{I.9})$$

where  $P(0)$  is simply the power measured at zero path length difference. We can rearrange this expression and define the power function  $W(x)$

$$W(x) = 2P(x) - P(0) = \int_{-\infty}^\infty G(k) e^{ikx} dk. \quad (\text{I.10})$$

We see then that the power function  $W(x)$  is the *inverse* Fourier transform of the power spectral density  $G(k)$ . Inverting this expression we have that the power spectrum is the Fourier transform of the power function

$$G(k) = \frac{1}{2\pi} \int_{-\infty}^\infty W(x) e^{-ikx} dx, \quad (\text{I.11})$$

or equivalently

$$G(\nu) = \frac{1}{2\pi} \int_{-\infty}^\infty W(x) e^{-i2\pi\nu x/c} dx. \quad (\text{I.12})$$

### Analysis with MATLAB

In order to compute the power spectra from your fringe pattern data, you will need to do some data processing. First, you will need to properly truncate your data because the vector of data, `data`, may include points when the stage was

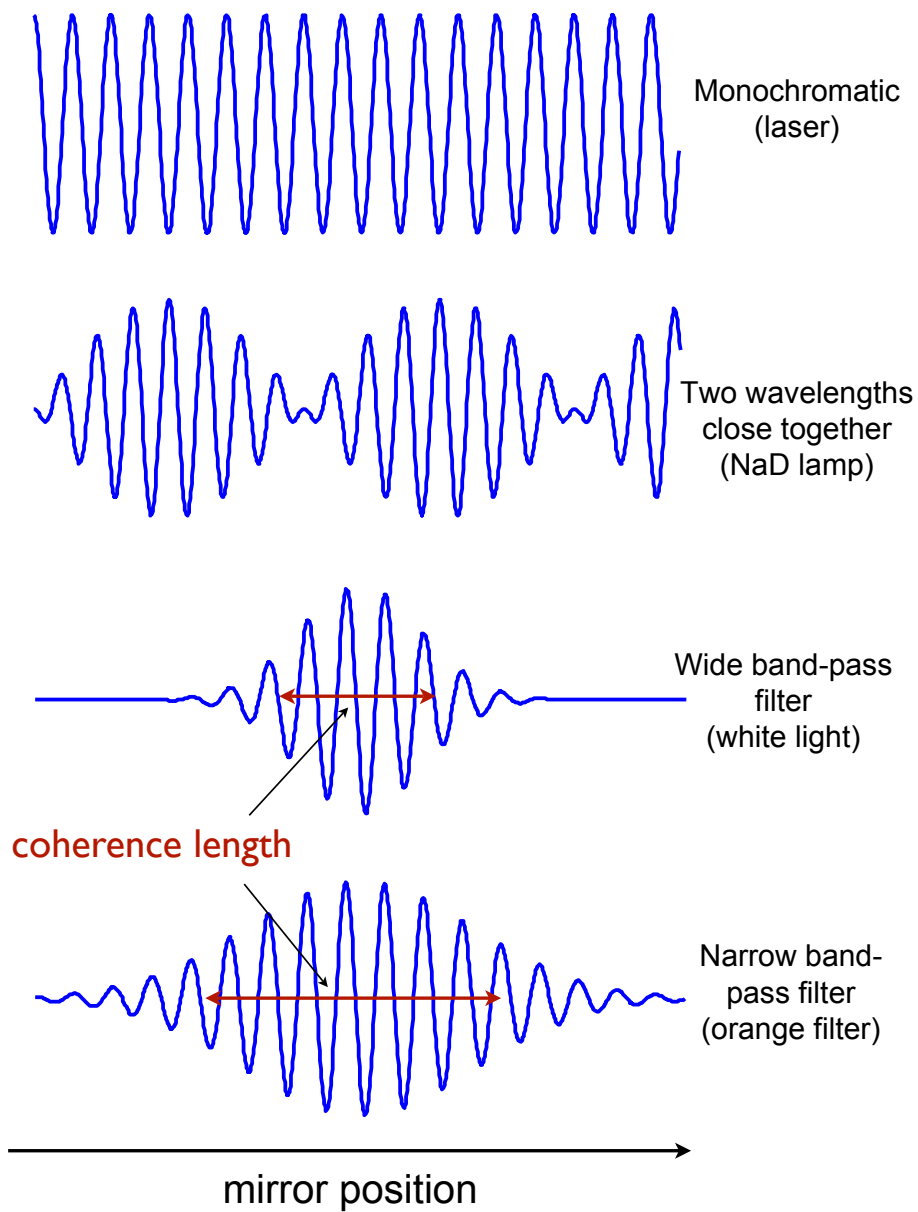


Figure I.1: Fringe patterns plotted as a function of the mirror position. Notice how the fringe visibility collapses and then revives as a function of the mirror position when the spectrum is discrete. Also, note that if the mirror is moved a distance  $d$ , the optical path length between the two arms in the interferometer change by  $2d$ .

moving backwards and then sitting stationary before and after the slow sweep through the ZPL position. Therefore, find the points AA and BB which are at the beginning and end of the data and truncate the data. Your code will look something like the following:

```
>> rawdata = michelson_timestream(span, speed);
>> data = rawdata(AA:BB);
```

Inspect a plot of your data to check that it has been properly truncated. Now, you should transform your vector of data into something equivalent to  $W(x)$  (well equivalent up to a constant and a scale factor) by subtracting off the mean and then normalizing the data vector with

```
>> ndata = data-mean(data);
>> ndata = ndata/(max(ndata));
```

Now, to take the Fourier transform, you will use the “Fast Fourier Transform” function (`fft`) which, because of the structure of this super-efficient numerical algorithm, requires that you feed it a vector whose length is equal to a power of 2. Fortunately you can feed the function your vector and tell it the length is longer by rounding up to the next higher power of 2 (and the function will automatically pad your data with zeros at the end to fill in the gap). for this, use some code like the following:

```
>> L = length(ndata);
>> NFFT = 2^nextpow2(L);
>> fftdata = abs(fftshift(fft(ndata,NFFT)));
>> fftdata_oneside = fftdata(NFFT/2+2:NFFT);
>> nfftdata_oneside = fftdata_oneside/max(fftdata_oneside);
```

This will give you the absolute value of the Fourier transform since we aren’t interested in the phase factors, only the amplitude versus wavelength. Also, we are only interested in the positive frequency components of the FT, so we will plot `fftdata_oneside`. The last line just normalizes the result by the max value. Why are the array limits `NFFT/2+2` and `NFFT` for the command “>> `fftdata_oneside = fftdata(NFFT/2+2:NFFT)`;”? The reason is that the  $(N/2+1)$  element is the zero frequency bin and the  $(N/2)$  element is actually one of the negative frequency bins. The positive frequencies start at  $(N/2 + 2)$ .

Now, the data contained in `fftdata` are the values of  $|G(k)|$  (technically  $|G_n|$  since we just have a list of values indexed by  $n$ ) where the sample size is  $\Delta_k = \frac{2\pi}{N_s \Delta_x}$ .  $N_s = \text{NFFT}$  is the number of samples in the data which was “Fast Fourier Transformed” and  $\Delta_x$  is the step size in, for example, nanometers between images. Remember since  $\Delta_x$  is defined as the amount the optical path length varied between frames, it is twice the amount the carriage moved between images.

So, you know that each value of  $|G_n|$  is paired with a wavevector of  $k = n \times \Delta_k$ . But you should plot your spectral data in terms of wavelength  $|G(\lambda)|$  and so you need to create a vector of  $\lambda$  values corresponding to each point in  $|G_n|$ . Using the fact that  $k = \frac{2\pi}{\lambda}$ , we can write that  $\lambda = \frac{2\pi}{n\Delta_k}$

```
>> n=1:1:NFFT/2-1;
>> lambda=zeros(size(fftdata_oneside));
>> lambda=NFFT*delta_x./n;
>> plot(lambda,fftdata_oneside);
```



The little dot (.) at the end of `delta_x`. insures that the division to create the vector `lambda` from the vector `n` happens element-wise.

**An alternative** and more transparent example is the following (given your data file has an EVEN number of elements):

```
>> data = rawdata(AA:BB);
>> data = data - mean(data);
>> N = length(data);
>> spec = abs(fft(data));
```

Now we remove (or just leave out) the zero frequency at index 1, and 'negative' frequencies ( $N/2+2:N$ )...

```
>> spec = spec(2:N/2+1);
>> spec = spec / max(spec);
```

And, since we've removed zero frequency, the `lambda` index starts at 1.

```
>> n = 1:N/2;
>> lambda = N*delta_x./n;
>> plot(lambda, spec);
```



## Appendix J

# Michelson Interferometer MATLAB Scripts

This appendix provides a brief description of the two MATLAB scripts that are used in the Michelson Interferometer lab. The files are located in the desktop folder “Michelson files”. Do not run the **WinTV2000** program at the same time as running any of these programs as it will not allow MATLAB to access the video card. The comments of the M-files will contain more specific information about each of the files.

### **michelson\_timestream**

When this script is executed, it will prompt you to click on the image the location where the fringe pattern is the most visible. The script will then move the stage back by a distance (**span**) specified in mm, then it will move the stage forward by twice that distance (**2 x span**) at a slow velocity (**speed**) specified in mm/s, and then it will move the stage back to the starting point. To use this script, your code will look like this

```
>> data = michelson_timestream(span, speed);
```

where **span** and **speed** are to be chosen by you, and the data will be returned into the array **data**. As an example, consider this code

```
>> data = michelson_timestream(0.01, 0.001);
```

This will move the stage back by  $10\ \mu\text{m}$  and then forward by  $20\ \mu\text{m}$  at a speed of  $1\ \mu\text{m/s}$  (which will take 20 seconds), and then back  $10\ \mu\text{m}$  to the starting point. The data will be returned into the array **data** which will have the intensity on the pixel you selected for each frame of the video. Since the video frame rate is 30 frames per second, this means the stage will have moved  $\frac{1\ \mu\text{m/s}}{30\ \text{frames/s}} = \frac{33\ \text{nm}}{\text{frame}}$ .

### **vid\_capture**

For the index of refraction of air measurements, you can use the **vid\_capture MATLAB** script to capture the intensity on a small area in the CCD image as

a function of time. Your code will look like this “>> d = vid\_capture(time);” where you choose `time` appropriately (try 20 seconds).

To do your measurement, you should first prepare a vacuum in the cell: close the “leak valve” and open the “pump valve” and turn on the vacuum pump. After the pressure in the cell (as read by the meter) has dropped to the minimum on the scale, close the “pump valve” and turn off the pump. Start the data acquisition and slowly open the “leak valve” to let in air while you count fringes. You may need some practice introducing the air fast enough that the pressure rises to atmospheric pressure during the `time` you have set while not rising too quickly so that you miss the passage of fringes due to the finite sampling time of the video capture. Be careful to insert the glass vacuum cell without hitting or misaligning the mirrors.

## Appendix K

### Image Formation

In the figure below a mesh object is illuminated with collimated coherent radiation (produced by a laser) and a magnified image of the mesh is formed by the second lens, referred to as the transform lens. The magnified image is located in the plane a distance  $s_i$  away from the lens (at position  $z = B$ ) which is itself a distance  $s_o$  from the object. In the thin lens approximation, the magnification is equal to the ratio

$$m = \frac{s_i}{s_o}, \quad (\text{K.1})$$

and the object and image distances ( $s_o, s_i$ ) are related to the focal length  $f$  by the image equation

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f} \quad (\text{K.2})$$

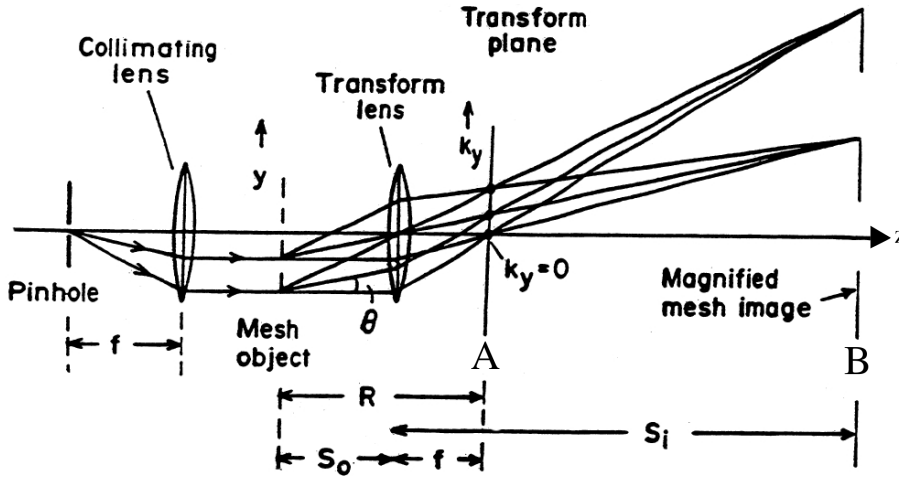


Figure K.1: Magnified image formation with parallel coherent incident radiation.

In contrast, if the screen is placed at  $z = A$ , something else is produced. In

particular, as will be shown below, the spatial 2-D Fourier transform<sup>1</sup> of the object image will appear in the plane at  $z = A$

### Optical Fourier Transform Produced by a Lens

In order to understand how a lens generates the Fourier Transform of the light wave emanating from the object, let's imagine that we break up this object wave into a superposition of plane waves (this can always be done for linear optical systems). Now, the lens simply focuses each of these plane waves to a different spot in the focal plane. So, the optical wave at the focal plane behind the lens is the 2-D Fourier transform of the optical wave leaving the object.

Let's see how this works. Consider a plane wave of complex amplitude  $U(x, y, z) = A \exp[i(k_x x + k_y y + k_z z)]$  with wavevector  $\mathbf{k} = (k_x, k_y, k_z)$ , wavelength  $\lambda$ , wavenumber  $k = (k_x^2 + k_y^2 + k_z^2)^{1/2} = 2\pi/\lambda$ , and complex envelope  $A$ . The vector  $\mathbf{k}$  makes angles  $\theta_x = \sin^{-1}(k_x/k)$  and  $\theta_y = \sin^{-1}(k_y/k)$  with the  $y - z$  and  $x - z$  planes, respectively, as illustrated in Fig. ???. The complex amplitude in the  $z = 0$  plane,  $U(x, y, 0)$ , is a spatial harmonic (i.e. sinusoidal) function  $f(x, y) = A \exp[2i\pi(v_x x + v_y y)]$  with spatial frequencies  $v_x = k_x/2\pi$  and  $v_y = k_y/2\pi$  (cycles/mm). The angles of the wavevector are therefore related to the spatial frequencies of the harmonic function by

$$\begin{aligned}\theta_x &= \sin^{-1} \lambda v_x \\ \theta_y &= \sin^{-1} \lambda v_y\end{aligned}\tag{K.3}$$

Recognizing  $\Lambda_x = 1/v_x$  and  $\Lambda_y = 1/v_y$  as the spatial periods of the harmonic function in the  $x$  and  $y$  directions, we see that the angles  $\theta_x = \sin^{-1}(\lambda/\Lambda_x)$  and  $\theta_y = \sin^{-1}(\lambda/\Lambda_y)$  are governed by the ratios of the wavelength of light to the period of the harmonic function in each direction. These geometrical relations follow from matching the wavefronts of the wave to the periodic pattern of the harmonic function in the  $z = 0$  plane, as illustrated in Fig. ???.

If  $k_x \ll k$  and  $k_y \ll k$ , so that the wavevector  $\mathbf{k}$  is paraxial, the angles  $\theta_x$  and  $\theta_y$  are small (therefore  $\sin \theta_x \approx \theta_x$  and  $\sin \theta_y \approx \theta_y$ ) and

$$\begin{aligned}\theta_x &\approx \lambda v_x \\ \theta_y &\approx \lambda v_y\end{aligned}\tag{K.4}$$

Thus the angles of inclination of the wavevector are directly proportional to the spatial frequencies of the corresponding harmonic function.

Now, the different plane-wave components that constitute a wave may be spatially separated (i.e. isolated) by the use of a lens. Recall that a thin spherical lens transforms a plane wave into a paraboloidal wave focused to a point in the lens focal plane. If the plane wave arrives at small angles  $\theta_x$  and  $\theta_y$ , the paraboloidal wave is centered about the point  $(\theta_x f, \theta_y f)$ , where  $f$  is the focal length (see Fig. ???). The lens therefore focuses (maps) each plane wave propagating in the direction  $(\theta_x, \theta_y)$  onto a single point  $(\theta_x f, \theta_y f)$  in the focal plane and thus spatially separates the contributions of the different harmonic functions.

---

<sup>1</sup>If you don't remember what a Fourier transform is, now is a good time to review it. Also see <http://www.falstad.com/fourier/> for an instructional java applet.

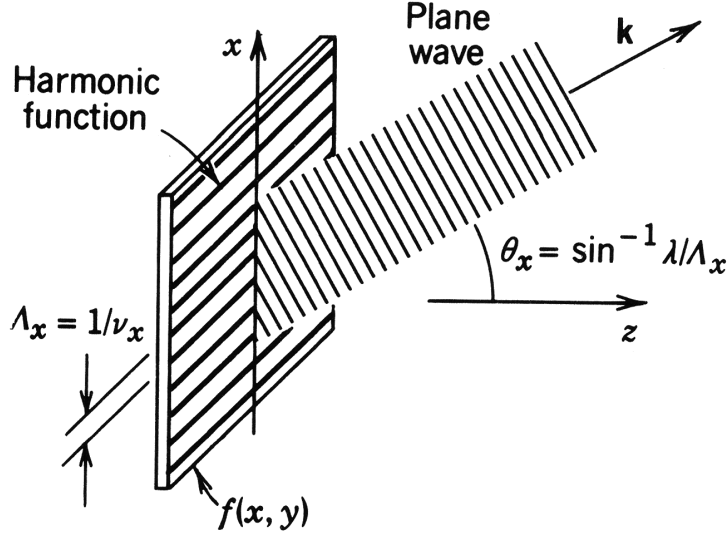


Figure K.2: A harmonic function of spatial frequencies  $v_x$  and  $v_y$  at the plane  $z = 0$  is consistent with a plane wave traveling at angles  $\theta_x = \sin^{-1} \lambda/\Lambda_x$  and  $\theta_y = \sin^{-1} \lambda v_y$ .

In reference to the optical system shown in Fig. ??, let  $f(x, y, z)$  be the complex amplitude of the optical wave in the  $x$ - $y$  plane at some position  $z$ . The optical wave emanating from the object in the  $z = 0$  plane located to the left had side of the lens is then  $f(x, y, 0)$ . Think of the light coming from the object as a superposition of plane amplitude waves, with each wave component traveling at a small angle  $\theta_x = \lambda v_x$  and  $\theta_y = \lambda v_y$  having a complex amplitude proportional to the Fourier transform of  $f(x, y, 0)$ , namely  $F(v_x, v_y)$ . Each of these plane wave component is then focused by the lens to a unique point  $(x', y')$  in the focal plane where  $x' = \theta_x f = \lambda f v_x$ ,  $y' = \theta_y f = \lambda f v_y$ . The complex amplitude of the wave at point  $(x', y')$  in the output or focal plane (located at  $z = A$ ) is therefore proportional to the Fourier transform of  $f(x, y, 0)$  evaluated at  $v_x = x'/\lambda f$  and  $v_y = y'/\lambda f$ . Thus we have

$$f(x, y, z = A) \propto F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right) \quad (\text{K.5})$$

Short range irregularities in  $f(x, y, 0)$  (sharp edges) contribute primarily to the diffraction field away from the optic axis, *i.e.* for large  $(x', y')$  or  $(k_x, k_y)$ . Long range irregularities (soft edges) contribute primarily to the small spatial frequencies  $(k_x, k_y)$  near the optic axis in the diffraction pattern. Obviously, spatial filtering of the beam is performed in the transform plane. This filtering can be of simple low-pass or high-pass type, or it can involve selective phase-change of any particular spatial frequency component. Later we will investigate the filtering properties.

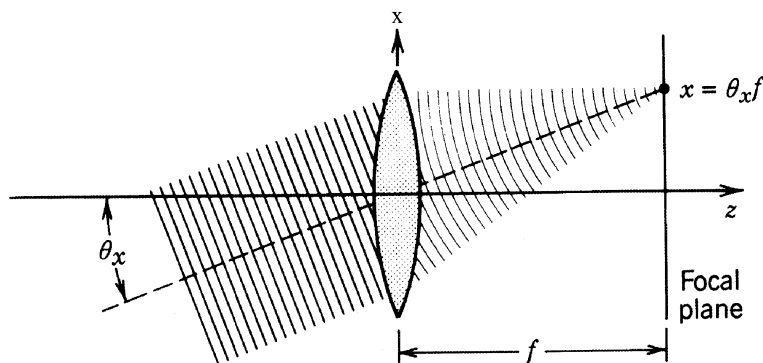


Figure K.3: Focusing of a plane wave into a point. A direction  $(\theta_x, \theta_y)$  is mapped into a point  $(x, y) = (\theta_x f, \theta_y f)$  at  $z = f$ .

### Inverse Fourier Transform

So if the Fourier transform appears at  $z = A$ , how does an image appear at  $z = B$ ? Very simply, propagation of the wave through free space from  $z = A$  to  $z = B$  generates the inverse Fourier transform! In this way, the object wave is modified by the lens so that propagation of the wave after the lens produces the Fourier transformed at the transform plane  $z = A$ , and then, by additional propagation through free space, the inverse Fourier transform is created at  $z = B$ . This process is what produces an image of the object at  $z = B$ .

To see how propagation generates the inverse transform, consider this: the electromagnetic wave which crosses the plane at  $z = A$  can be thought of as originating from a distribution of point emitters in that plane, and each of those point emitters illuminates the image plane (at  $z = B$ ) after propagation by the distance  $d$  between  $A$  and  $B$ . If you write the image wave at  $z = B$  as the sum of spherical waves generated by all these point emitters, you get something which, after some approximations, is exactly the inverse Fourier Transform!

Here's the same argument but with the math too. First consider just a single point source of light located on the  $z$ -axis in the Transform plane at  $z = A$ . This point source generates a spherical wave propagating outward (i.e.  $E = \frac{1}{r} e^{i(kr - \omega t)}$  where  $r$  is the length of the vector pointing from the point source to the observation point). If we examine this spherical wave far from the source but close to the  $z$  axis, we can safely approximate it as a paraboloidal wave (i.e.  $E \simeq \frac{1}{z} e^{ikz} e^{ik(x^2 + y^2)/2z}$ ). If the point source emitter weren't located exactly *on* the  $z$ -axis (i.e. at  $x = 0, y = 0$ ) but rather just off the  $z$ -axis, say at  $(x_i, y_i)$ , then we simply replace  $x \rightarrow x - x_i$  and  $y \rightarrow y - y_i$  since the position variables come from the vector pointing from the source at  $\vec{r}_i$  to the observation point at  $\vec{r}$ . In this case, the electric field produced in the plane at  $z = B$ , a distance  $d$  from the point emitter at  $z = A$  is then

$$E(x, y, z)|_{z=B} \simeq \frac{1}{d} e^{ikd} e^{ik((x-x_i)^2 + (y-y_i)^2)/2d} \quad (\text{K.6})$$

Okay, now if we have a whole distribution of point sources of different amplitudes (brightnesses) in the plane  $z = A$  described by the function  $E(x, y, z = A)$ , then



the electric field observed in the plane  $z = B$  far from  $z = A$  will just be the sum of all these point sources each located at some initial position  $(x_i, y_i)$  and with some amplitude  $E(x_i, y_i, z = A)$ . This sum is just the integral

$$E(x, y, z)|_{z=B} \simeq \frac{1}{d} e^{ikd} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_i, y_i, z)|_{z=A} e^{ik((x-x_i)^2 + (y-y_i)^2)/2d} dx_i dy_i. \quad (\text{K.7})$$

This equation expresses the form of the diffracted field after propagation by a distance  $d$ . In the far field limit this reduces to the Fraunhofer diffraction expression

$$E(x, y, z)|_{z=B} \simeq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_i, y_i, z)|_{z=A} e^{-i2\pi(x x_i + y y_i)/\lambda d} dx_i dy_i. \quad (\text{K.8})$$

where  $k$  has been replaced by its value  $2\pi/\lambda$ , we have dropped the unimportant phase prefactor  $(\frac{1}{d} e^{ik(d+[x^2+y^2]/2d)})$  which doesn't matter if we only look at the field amplitude. Also, we have dropped the quadratic terms inside the exponent  $(kx_i^2/2d)$ . This last step is okay so long as we're only interested in the field generated by point emitters lying close to the  $z$ -axis such that  $kx_i^2/2d \ll \pi$ . After these simplifications, we are left with just the integral which is the inverse Fourier Transform of the wave at  $z = A$ . The result is that if we modify the wave at  $z = A$  by placing there a mask, the image which results at  $z = B$  will be the convolution of the original image and the Fourier transform of the mask.

The Abbé theory of image formation arose from Abbé's consideration of these effects when trying to view small objects through a microscope. It became clear to him that a lens (the objective lens of the microscope) always acts as a low-pass filter, since its finite size will admit only spatial frequencies up to an obvious geometrical limit. The bigger the lens and the larger the range of spatial frequencies admitted, the greater the image fidelity. In the limit of admitting only the undeflected central order ( $\theta = 0$ ), no detail of the object at all will be present in the image. Only a uniform illumination (the so-called d.c. level) will be obtained in the image plane. This, therefore determines the size of the smallest objects which can be viewed by a particular microscope. This relationship is exploited to "clean" up the laser beam coming from the HeNe laser. By focussing the beam through a pinhole, we spatially filter out the high spatial frequencies (i.e. the transverse spatial structures) of the laser to give a clean uniform beam without structure.

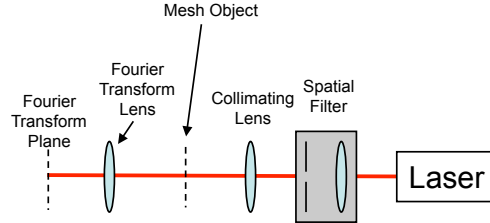


Figure K.4: Part of the imaging setup for Fourier Optics lab. The spatial filter and collimating lens expand and collimate the laser beam which then illuminates the mesh object. The optical wave transmitted by the mesh passes through the Fourier Transform lens and at the focal distance ( $f$ ) after that lens, the Fourier transform of the transmitted beam appears. A picture of the mesh Fourier transform intensity pattern is shown in Fig. ??

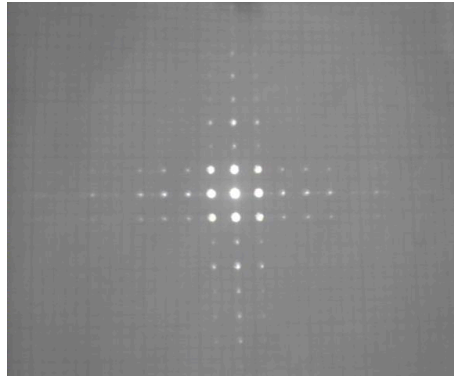


Figure K.5: Picture of the intensity profile at the Fourier Transform plane produced by a wire mesh object. See Fig. ?? for the optical setup.

## Appendix L

# Concave and Convex Lenses

Converging lenses, which are generally thicker at the center than at their margins, focus collimated light (a parallel beam) to a point in their rear, or secondary focal plane. Diverging lenses are wider at the perimeter and "focus" collimated light in their rear focal plane (see ??). This is the definition of a lens' focus.

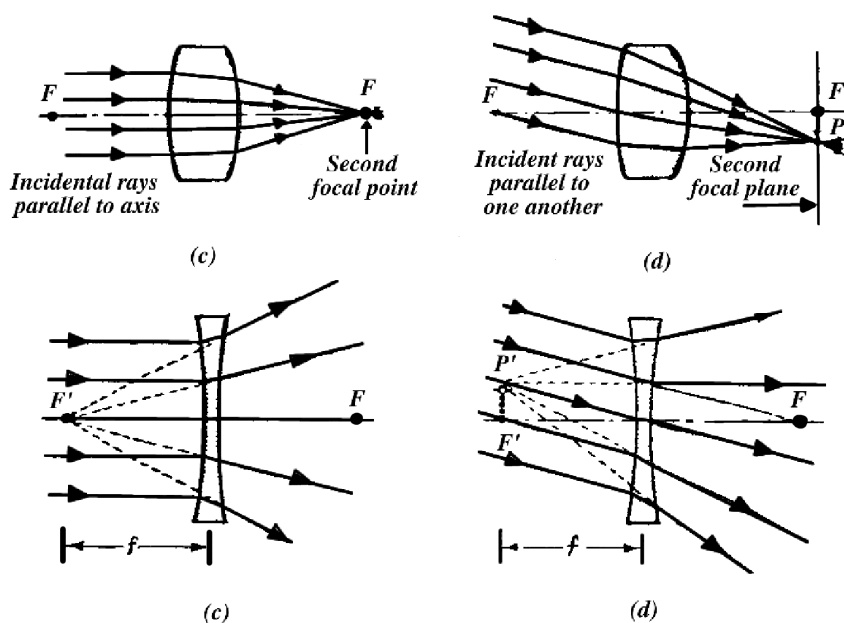


Figure L.1: Focal planes of converging and diverging lenses.

Images that are produced on the side of the lens opposite to the object are termed real because they can be seen on a screen. Images on the same side of the lens as the object are called virtual, and are seen only by looking through the refractive surface (from the opposite side, of course). Diverging lenses always produce virtual images, but a converging lens will produce either depending on the position of the object. In this experiment, we concern ourselves only

with real images. From a simple lens, real images are usually inverted, whereas virtual images are upright (see ??).

## Appendix M

# Phase-Contrast Imaging

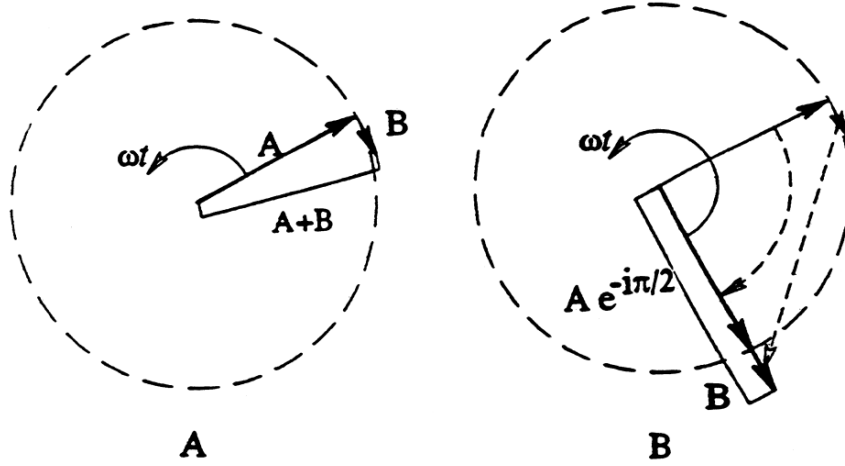


Figure M.1: Phasor diagram illustrating the principle underlying phase contrast imaging. In the left image (A) we see the two parts of the field  $\mathbf{A}$  and  $\mathbf{B}$  adding. In the text, the field is  $f(x, y) = 1 + i\theta(x, y)$ , so here  $\mathbf{A} = 1$  and  $\mathbf{B} = i\theta$ . Because one is real and the other imaginary, they are orthogonal and the length of the resulting vector is almost the same length as the original  $\mathbf{A}$ . Therefore the contribution from  $\mathbf{B}$  is hard to see. In the right image (B), The  $\mathbf{A}$  component is first phase shifted by  $90^\circ$  and then added to  $\mathbf{B}$ . In this case, they add in the same direction, and any changes in  $\mathbf{B}$  produce a much larger change in the length of the resulting vector (the intensity of the field) than before when the two components were orthogonal. This increases the intensity contrast produced by the variation of the  $\mathbf{B}$  component making it more visible. Phase contrast imaging is therefore especially useful for seeing clear objects, such as a fingerprint on glass, where the object encodes its presence into the *phase* of the light.

The only spatial filtering discussed so far has involved altering the amplitude of the transmission function, by passing only selected spatial frequencies through

the Fourier Transform plane. However, the phase of the light wave is also available at the Fourier Transform plane, and altering the phase of selected spatial frequencies using phase changing spatial filtering masks offers different possibilities for optical image manipulation and reconstruction.

Consider biological imaging applications where the objects you are trying to observe (say a protist swimming in water) are almost as transparent as the medium in which they are situated. In addition, different parts of the cell will typically have the same absorption coefficient and therefore result in the same output wave amplitude, but they have different indices of refraction. As a result, a substantial amount of information is encoded in the *phase* of the light. However, because photodetectors (like your eye) measure the intensity of the wave, this phase information is lost in a normal microscope. Phase contrast imaging is a technique by which spatial filtering is used to transform the phase variation across the wave into amplitude variations which can then be detected. The invention of this method was first applied in Phase-Contrast Microscopy, which earned Fritz Zernike a Nobel Prize.

Returning our attention to the protist, the index of refraction of this creature is close to that of water, so the transmission function will be approximately equal to that of water but with a small position dependent phase rotation (see figure ??A). A plane wave passing through the water will have the form  $e^{ikd}$  where  $d$  is the thickness of the water layer. This same plane wave will pick up an extra phase term from the modification of the transmission function due to the protist in the water. The complex amplitude of the plane wave passing through the protist will be  $f(x, y) = e^{ikd}e^{i\theta(x, y)}$ , where  $\theta(x, y)$  is the position dependent additional phase shift induced by the protist body and its organelles. If we were to image this wave on a detector, the intensity profile,  $I(x, y) \propto |f(x, y)|^2 = 1$ , is completely uniform and we can't possibly see the creature like this. So, we need to implement phase contrast imaging to reveal the phase information. Let's see how this works.

Assuming the additional phase shift is small,  $\theta(x, y) \ll 1$  (this assumption is not so important but makes the math less messy), the exponent can be approximated as  $e^{i\theta} \simeq 1 + i\theta$ . Dropping the overall phase of the plane wave in  $f(x, y)$ , we have

$$f(x, y) \simeq 1 + i\theta \quad (\text{M.1})$$

$$|f(x, y)|^2 \simeq 1 + i\theta - i\theta - i^2\theta^2 = 1 + \theta^2 \quad (\text{M.2})$$

Under this approximation, we can see that (again) there is no information showing up in the square of the field (intensity), since  $\theta \ll 1$ . So, what should we do? What would happen if we first changed the "1" in " $1 + i\theta$ " to an " $i$ "? This would correspond to a  $90^\circ$  phase shift of the DC term (the "1") while leaving the spatially varying part (the  $\theta$ ) unchanged (See Figure ??B). In this case we have

$$|i + i\theta|^2 = -i^2 - i^2\theta - i^2\theta - i^2\theta^2 \quad (\text{M.3})$$

$$= 1 + 2\theta + \theta^2. \quad (\text{M.4})$$

Now the intensity has a  $2\theta$  term which is still smaller than the DC term but much larger than  $\theta^2$  and would give enough contrast against the flat illumination of the plane wave to see the protist. So, how do we affect this selective  $90^\circ$  phase shift of the DC term?

Since the DC term has no spatial variation, representing just a flat and featureless illumination from the plane wave, it will be focused to a point in the Fourier Transform plane and comprise the zeroth order of the diffraction pattern. This suggests a simple way to alter the phase of the uniform illumination, just insert a thin dielectric layer (film) at the central spot in the transform plane. A quarter wavelength phase change would produce the best results (turning the “1” into an “ $i$ ”), which is what Zernike did, but clearly any phase change would help us distinguish between the object and the water. Commercial phase contrast microscopes often have an adjustable phase.

### Experimental Arrangement

The grating is manufactured so that the phase shift acquired by the light passing through it varies across the surface. Such a grating can be realized by etching the glass to produce parallel ridges and troughs (See Figure ??). The added path length in glass produces a small phase difference between the ridges and troughs. However, the image on the screen should look featureless (like it would for a glass plate) as the ruling doesn’t change the amplitude of the wave and the phase differences are very small. Ensure that the image is properly in focus at which point it should show no signs of the grating.

***PLEASE DO NOT TOUCH THE GRATING SURFACE !***

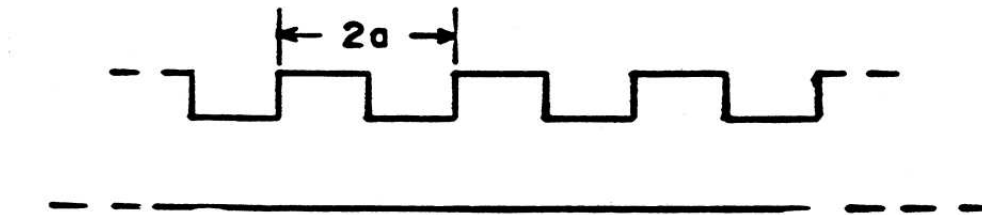


Figure M.2: Cross-section of the transparent phase-grating.





# Appendix N

## Diffraction

The term diffraction is generally understood to mean the propagation of light “around” the edges of obstacles or apertures. Diffraction phenomena are traditionally divided into two classes - Fresnel and Fraunhofer diffraction - although they are both manifestations of the same phenomenon. Fraunhofer diffraction refers to what happens to an optical pattern after propagation over a very large distance  $z$  in the paraxial approximation. Fraunhofer diffraction is the *far field* limiting case of Fresnel diffraction that arises when both source and observer are at very large distances from the diffracting screen such that the incident and diffracted waves are effectively plane. In this limit, the curvatures of the incident and diffracted waves can be safely neglected and the Fraunhofer diffraction pattern is just the Fourier transform of the initial pattern. If, however, either the source or receiving point is close to the diffracting aperture such that the wave front curvature is significant, then the result will be Fresnel diffraction [see Fig. ??<sup>1</sup>].

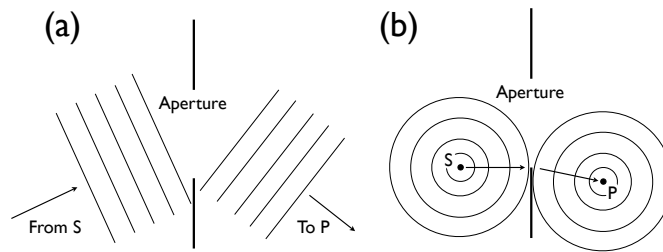


Figure N.1: Diffraction by an aperture in the (a) Fraunhofer limit and (b) Fresnel limit

In order to be in the Fraunhofer limit, the distances between the source and diffracting aperture and between the diffracting aperture and receiving point should be large compared to a characteristic distance,  $R_0$  given by;

$$R_0 = \frac{2w^2}{\lambda} \quad (\text{N.1})$$

---

<sup>1</sup>figure adapted from “Introduction to Modern Optics” by Grant R. Fowles

where  $\lambda$  is the wavelength of the light, and  $w$  is a dimension of the aperture or obstacle. This relation for  $R_0$  guarantees that the phase change across the width of the aperture is less than  $\frac{\lambda}{4}$ . Perfect Fraunhofer diffraction can only be achieved with collimated incident and diffracted beams.

### Diffraction Pattern of a Slit

In this experiment, you will investigate the two (Fraunhofer and Fresnel) diffraction regimes by studying the diffraction pattern from a slit aperture of variable width. You should start by inserting the collimating lens after the spatial filter so that the beam wave fronts are parallel. Thus the source of illumination is effectively at infinity. Place the slit after the collimating lens and reflect the transmitted light with the secondary mirror onto the screen with the image visible by the CCD camera.

The diffraction pattern observed can be characterized by a dimensionless parameter,  $\Delta v$ , defined as;

$$\Delta v = w \left( \frac{2}{R\lambda} \right)^{1/2} = \left( \frac{R_0}{R} \right)^{1/2} \quad (\text{N.2})$$

where  $w$  is the width of the slit, and  $R$  is the slit-screen distance. Fresnel diffraction occurs for  $\Delta v \geq 1$ . Fraunhofer diffraction, the so called far-field approximation is characterized by  $\Delta v \ll 1$ . This condition can be established with either a large slit-screen distance as the name implies, or a narrow slit. In the lab, the slit width is the more convenient adjustment to vary the  $\Delta v$  value, since a large range of  $w$  is easily achieved with a turn of the micrometer. However, varying  $R$  is equally acceptable. For each measurement you make, you should compute your  $\Delta v$  value on the spot to insure that you are in the proper limit (recall that the wavelength of the He-Ne laser is 632.8 nm). Because the Fraunhofer diffraction pattern is essentially the Fourier Transform of the input wave, the Fraunhofer diffraction can also be observed at the focal plane of a converging lens (see Figure ??)

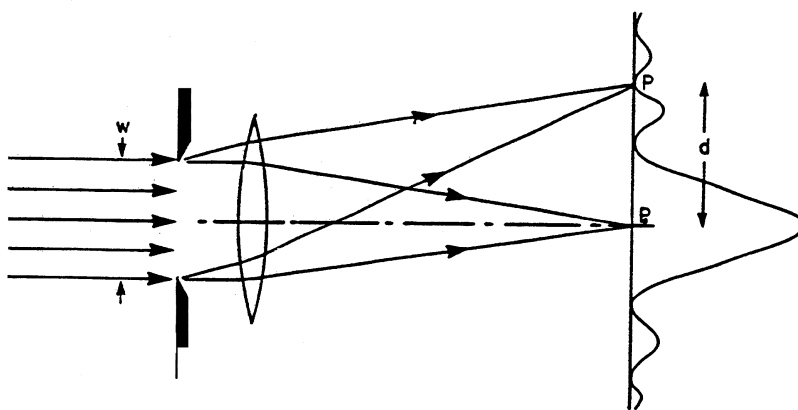


Figure N.2: Fraunhofer diffraction pattern of a slit formed at the focal plane of a lens. In this part of the lab, you will generate your Fraunhofer patterns *without* the aid of a “Fourier Transform” lens placed after the aperture.

Start by adjusting the slit width to be very narrow until you see the characteristic Fraunhofer pattern. Record and reproduce in your lab write-up the image of the observed diffraction pattern.

Now slowly widen the slit width watching the diffraction pattern as you do. You should be able to see the transition from Fraunhofer to Fresnel diffraction, with the initial loss of the zeros, and subsequent increase of secondary maxima. Record the image of the Fresnel patterns at values of  $\Delta v \simeq 0.5, 1$ , and 4. For each measurement you make, you should compute your  $\Delta v$  value to insure that you are in the proper limit. Be sure that the image is not saturated so that your data is good. To be safe, take several images of the same pattern at different laser intensities. There are two polarizers available to reduce the intensity of the laser so that the patterns are not saturated. Note: for some patterns, reducing the intensity so that the central maximum does not saturate the camera makes the “wings” of the pattern invisible. In this case, having two or more images at different intensities may be important. Also, the line out that you take of the image of the slit diffraction for comparison with theory can be chosen in a region where the image is not saturated.

Your images should look something like those in Fig. ?? . Since the CCD has an auto gain circuit, it may help to turn on the room lights so that the patterns are visible and not completely saturated. Your analysis will be done by taking a line out of the image, so as long as there is a part of the image that is not saturated, you can extract the data. Because the CCD has a limited dynamic range, the image may not show clearly the low intensity wings of the diffraction pattern that will be obvious to you by eye.

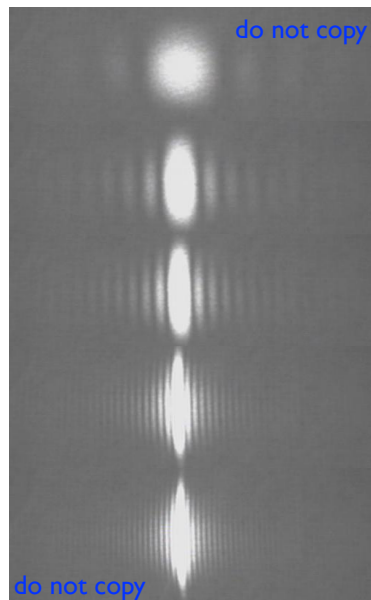


Figure N.3: Diffraction patterns from a slit of various widths.

### Analysis of the slit diffraction

Take your images and import them into an array in MATLAB, maple, mathematica, octave, or your favourite math tool. Plot the intensity as a function of position along an axis perpendicular to the slit axis (let's call that the  $x$ -axis, and this axis is horizontal in the images shown in Fig. ??). To improve the signal to noise of your data, you may need to integrate (add pixel intensities) the image along the  $y$  direction. Since there might be some distortion of the image along the vertical ( $y$ ) direction (as seen in Fig. ??), you should probably only sum up the 10 adjacent pixels along a vertical line for each sample along  $x$ . Be sure that you choose an area of the image that is not saturated. In Fig. ??, this area would correspond to the top of the images above the point where the central spot is completely saturated. The problem of saturation is probably most apparent in the lowest image where the interference fringe spacing is the smallest. Calculate your values of  $\Delta v$ , and compare your diffraction patterns to theory by plotting your experimentally measured data against the predicted diffraction patterns at the corresponding  $\Delta v$ . For this you should numerically evaluate the form of the diffraction intensity using Matlab or Mathematica using the theory presented.

It is convenient to represent the intensity of the diffraction pattern at a distance,  $d$  from the optic axis in terms of the dimensionless parameter,

$$z = \frac{d}{w} \quad (\text{N.3})$$

Thus the edges of the geometric shadow correspond to  $z = \pm 0.5$ .

In the Fraunhofer limit ( $\Delta v \ll 1$ ), the intensity variation in the diffraction pattern is given by the Fourier transform of a square function,

$$I(z) \propto (\Delta v)^2 \sin^2 \beta / \beta^2 \quad (\text{N.4})$$

where  $\beta = (z\pi/2)(\Delta v)^2$ . The pattern has a global maximum at  $\beta = 0$ , with subsidiary maxima on both sides. These maxima are separated by zeroes in the intensity for

$$\pm\beta = n\pi \text{ or } \pm z = \frac{2n}{(\Delta v)^2} \quad n = 1, 2, 3, \dots$$

Notice that the intensity maximum is proportional to  $(\Delta v)^2$  and therefore to  $w^2$ , while the width of the central maximum is proportional to  $(\Delta v)^{-2}$  or to  $w^{-2}$ .

For Fresnel diffraction, ( $\Delta v > 1$ ), the general expression for the intensity is given by;

$$I(z) \propto \left[ C^2(v) + S^2(v) \right]_{v_1}^{v_2} \quad (\text{N.5})$$

where  $C(v)$  and  $S(v)$  are the Fresnel cosine and sine integrals, respectively, given by

$$C(v) = \int_{v_1}^{v_2} \cos\left(\frac{\pi x^2}{2}\right) dx$$

and

$$S(v) = \int_{v_1}^{v_2} \sin\left(\frac{\pi x^2}{2}\right) dx$$

The limits on the variable of integration are

$$v_1 = -(z + 0.5)\Delta v \quad \text{and} \quad v_2 = -(z - 0.5)\Delta v$$

In the limit of small  $\Delta v$ , equation ?? reduces to equation ??.

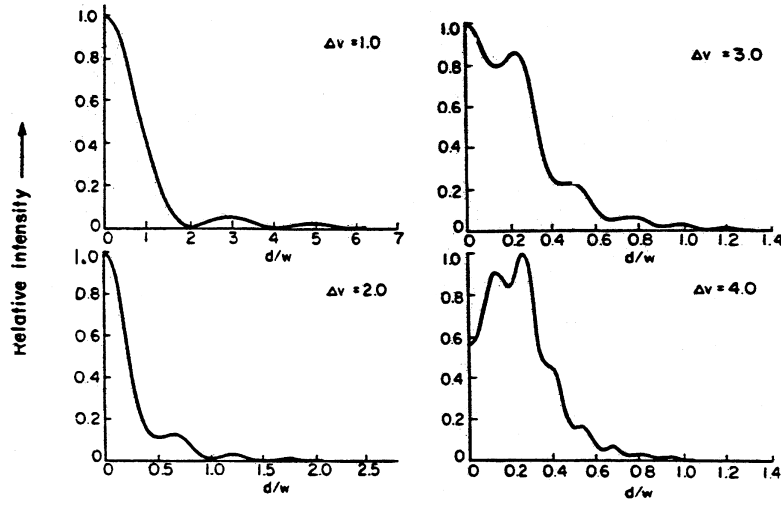


Figure N.4: Line intensities of Fresnel diffraction patterns of a slit for various values of the parameter  $\Delta v$ . Only one half of the symmetrical pattern is shown.