Problem set 2, due on October 20, **before** the beginning of the lecture

1 Magnetic moment in a more complicated situation (9 points)

In the previous problem you calculated the effective magnetic moment for the paramagnetic case of a spin-only $S = 1/2$ situation. From the lecture you know that there are more complicated configurations with a combined orbital and spin angular momentum, described by the angular momentum quantum number J, for which the magnetic quantum number can take many different values, namely $m_j = -J, -J+1, ...J$. We showed that from the Maxwell-Boltzmann distribution it follows

$$\langle m_z \rangle = \sum_{m_j=-J}^{J} -g\mu_B m_j \exp(-g m_j \mu_B H/(k_B T))/Z(T) \tag{1}$$

It can be shown that the sum results in

$$\langle m_z \rangle = gJ\mu_B \Phi_J(x) \tag{2}$$

where $x = gJ\mu_B H/(k_B T)$ and

$$\Phi_J(x) = \frac{2J+1}{2J} \coth\left(\frac{(2J+1)x}{2J}\right) - \frac{1}{2J} \coth\frac{x}{2J} \tag{3}$$

a) For the case that $x \ll 1$, show that $\Phi_J(x) \approx \alpha x$. What is the expression for α?

b) From this, derive an expression for $\langle m_z \rangle$. How does it depend on H and T?

c) If ρ is the number of paramagnetic atoms per unit volume, what is the magnetization per unit volume? Calculate the magnetic susceptibility $\chi = \mu_0 M/H$.