Problem set 4, due on October 13, **before** the beginning of the lecture

1 Continuity equations (group exercise, 10 points)

In the lecture you learned that electrical charges can be considered "sources" and "sinks" of **E**-field lines. Actually, the Gaussian law (first Maxwell equation) is an example of a continuity equation, which can be colloquially described as "You can't make something out of nothing".

a) Let us first deal with an example from hydrodynamics. Consider a fluid with the mass density $\rho(\mathbf{r}, t)$ flowing with the local velocity $\mathbf{v}(\mathbf{r}, t)$. The differential form of the continuity equation reads

$$\partial\rho/\partial t = -\nabla(\rho\mathbf{v}) \tag{1}$$

Using the divergence theorem (also known as Gauss' theorem, not to be confused with the electrodynamic Gaussian law), write this equation in its integral form and explain the meaning of the different terms.

b) Besides the first and second Maxwell equations, there is a further application of the continuity equation in electrodynamics: Consider a semiconductor which contains an electron distribution in the conduction band described by a charge density $\rho(\mathbf{r}, t)$ flowing with the local velocity $\mathbf{v}(\mathbf{r}, t)$. What is the physical meaning of the quantity $\rho\mathbf{v}$ in this case and which symbol did we use in the lecture for this quantity? In a semiconductor, electrons can also be "brought to" or "removed from" the balance by excitation of electrons into the conduction band (from the valence band or from intentionally doped impurity bands), or by the reverse process, the recombination of such electrons with empty spaces ("holes") left back during a previous excitation. Let $g(\mathbf{r}, t)$ be the excitation (generation) rate of electrons per volume (excitation density), and $h(\mathbf{r}, t)$ be the recombination rate. Write the corresponding generalized continuity equation, both in the differential and integral form, starting with an equation analogous to (1) but now including excitation and recombination into the balance. What is the meaning of the different terms in the integral form of the equation?

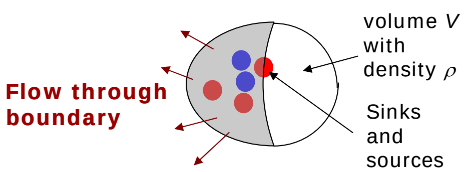

Figure 1: Schematic representation of a general physical situation where a continuity equation can be used. A physical quantity like a mass or a charge density within a given volume element changes by flux through the boundary of the volume. In a more generalized form, one can also allow "sources" or "sinks" within the volume.

2 Magnetic moment for a spin-1/2 material (8 points)

A paramagnetic material contains 10^{22} ions per cm^3, with a magnetic moment which is based on the spin angular momentum of the ion only, which in our example is assumed to be $S = 1/2$, i.e. the angular momentum quantum number is $J = 1/2$. Which values are then possible for the magnetic quantum number? For the case of a pure spin angular momentum, it can be shown that the g-factor g is 2.

a) A magnetic field of $B = 1$ Tesla is applied at $T = 300$ K. What is the population of the two resulting states, $p_\uparrow$ and $p_\downarrow$? Derive an expression for the excess population of the state with lower energy with respect to the one with higher energy $(p_\uparrow - p_\downarrow)/(p_\uparrow + p_\downarrow)$.

b) How large is the resulting magnetic moment $\langle m_z \rangle$ in Ampère per meter?