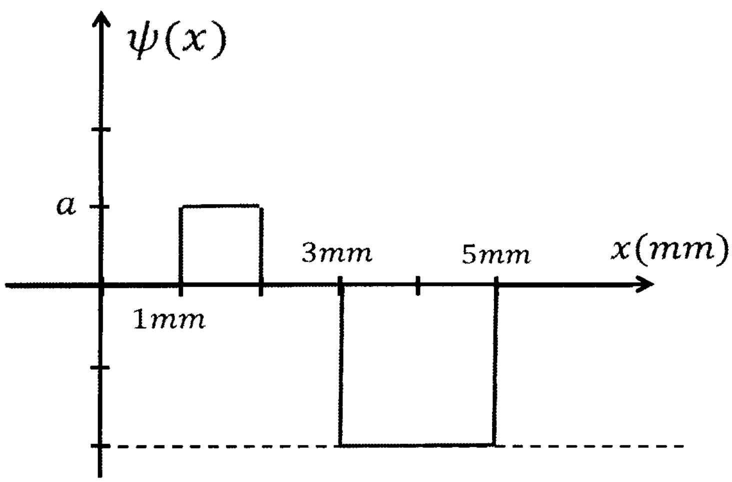
Tutorial: June 5th, 2009

Wavepacket representation of particles with Fourier series/transforms

The Davisson Germer experiment supports the DeBroglie hypothesis that electrons have wave-like properties – they show interference!

We introduced the wavefunction $\psi(x)$ and its interpretation, where $|\psi(x)|^2 dx$ represents the probability of finding the electron in the interval

You're now pretty good at solving problems like this:



1- What is the value for "a"?

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$$ Normalization

$$\int_{1mm}^{2mm} a^2 dx + \int_{3mm}^{5mm} (-2a)^2 dx = 1$$

$$a^2 (1mm) + 4a^2 (2mm) = 1$$

$$(9a^2) mm = 1$$

$$a = \frac{1}{3} mm^{-1/2}$$

2- If we make a measurement, what is the probability of finding the electron between 1mm and 2mm?

$|\psi|^2 dx$ is probability of finding e^- between x & $x+dx$

So $$\int_{1mm}^{2mm} |\psi(x)|^2\, dx = \int_{1mm}^{2mm} \underbrace{a^2}_{\frac{1}{9} mm^{-1}}\, dx = \frac{1}{9}$$

3- If we make a measurement of its position, what is the probability to find the electron at x=1mm?

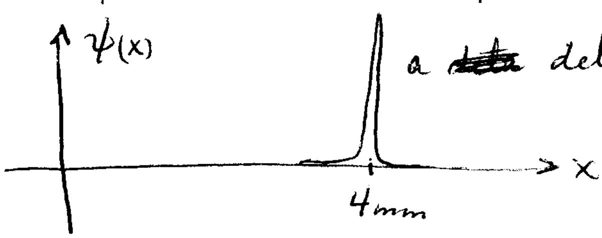
This interval Δx is $\underbrace{\text{zero}}_{\text{or infinitely small.}}$!

$$\int_{1mm}^{1mm} |\psi|^2\, dx = 0$$

4- What is the probability to find the electron between x=3mm and x=5mm?

$$\int_{3mm}^{5mm} |\psi(x)|^2\, dx = \int_3^5 (-2a)^2\, dx = (4\underbrace{a^2}_{\frac{1}{9} mm^{-1}})(2mm) = \frac{8}{9}$$

5- Suppose we make a measurement and find the electron at x=4mm...sketch the wavefunction of the electron *immediately* after the measurement. (hint: think of the photons or electrons hitting the screen in the double slit experiment. Make sure to ask for help from the TA's if you're struggling with this question)



a ~~data~~ delta function $\delta(x-4mm)$

Stop and go try out the simulation: "**Fourier, Making waves**"

Fine, but given $\psi(x, t=0)$ for a particle, how does it evolve in time? Does it even **change** as a function of time? If so, how?

<u>EM Waves (light/photons):</u>

Amplitude E = electric field

E^2 tells you probability of finding photon.

Maxwell's Equations:

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

Solutions are sin/cosine waves:

$$E(x,t) = A\sin(kx - \omega t)$$
$$E(x,t) = A\cos(kx - \omega t)$$

<u>Matter Waves (electrons, etc.):</u>

Amplitude ψ = "wave function"

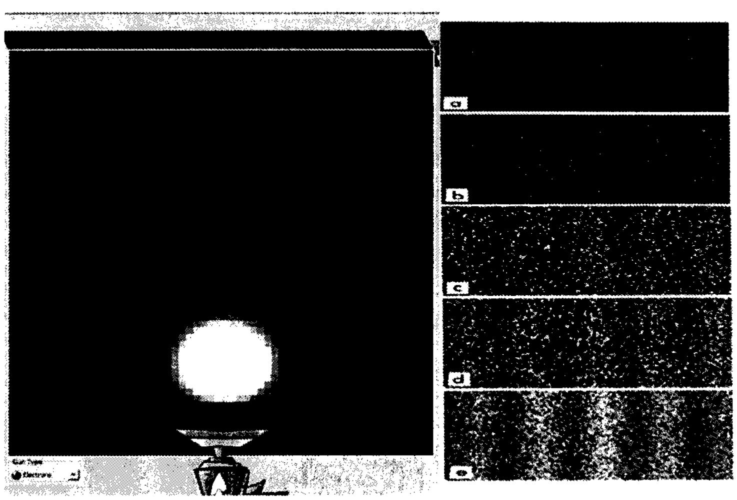
$|\psi|^2$ tells you probability of finding particle.

Schrodinger Equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t}$$

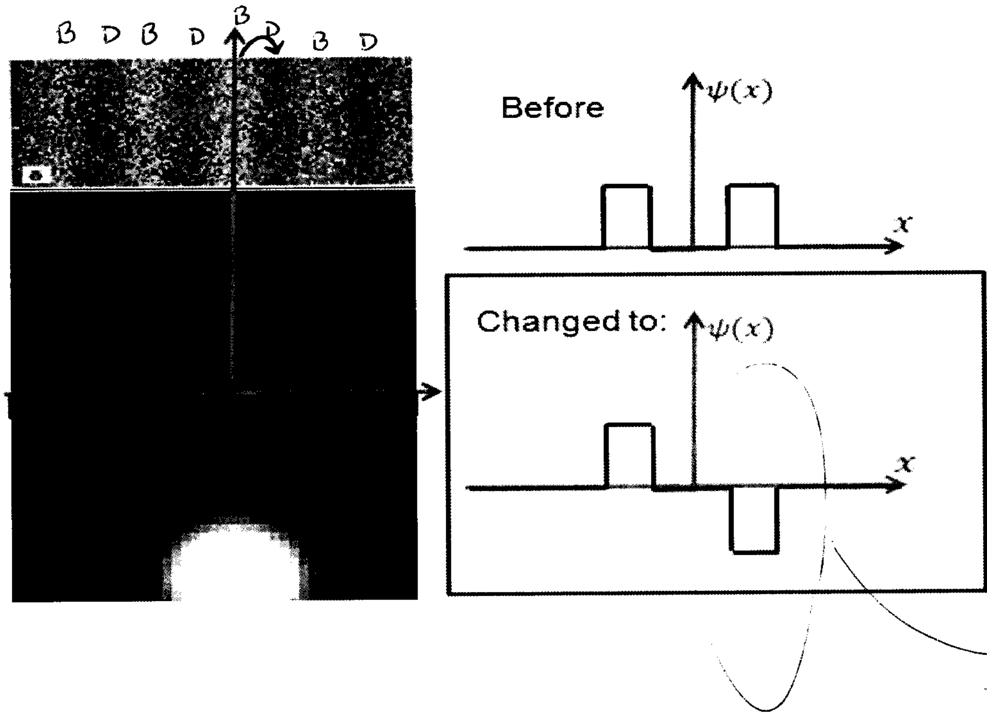
Solutions are complex sine/cosine waves:

$$\psi(x,t) = Ae^{i(kx-\omega t)} =$$
$$A(\cos(kx - \omega t) + i\sin(kx - \omega t))$$

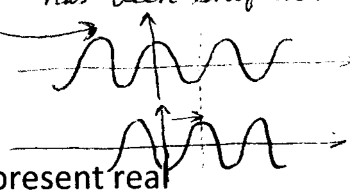


Clicker question: Do you expect the interference pattern to change?

(a) YES (b) NO *The central bright fringe becomes dark.*

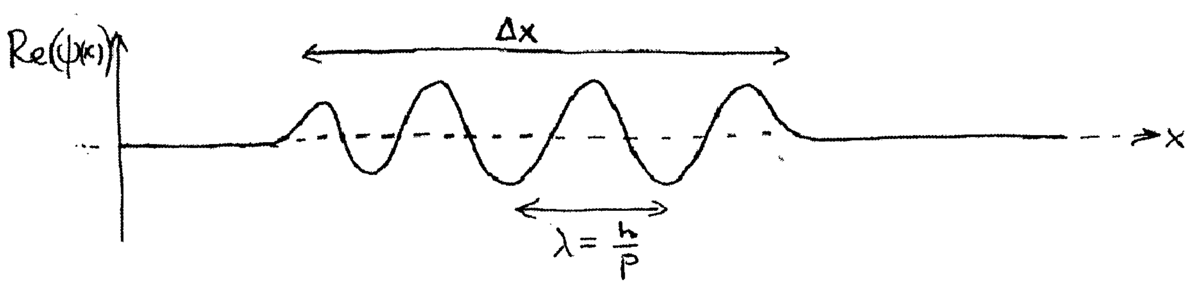


This wave contribution has been shifted.

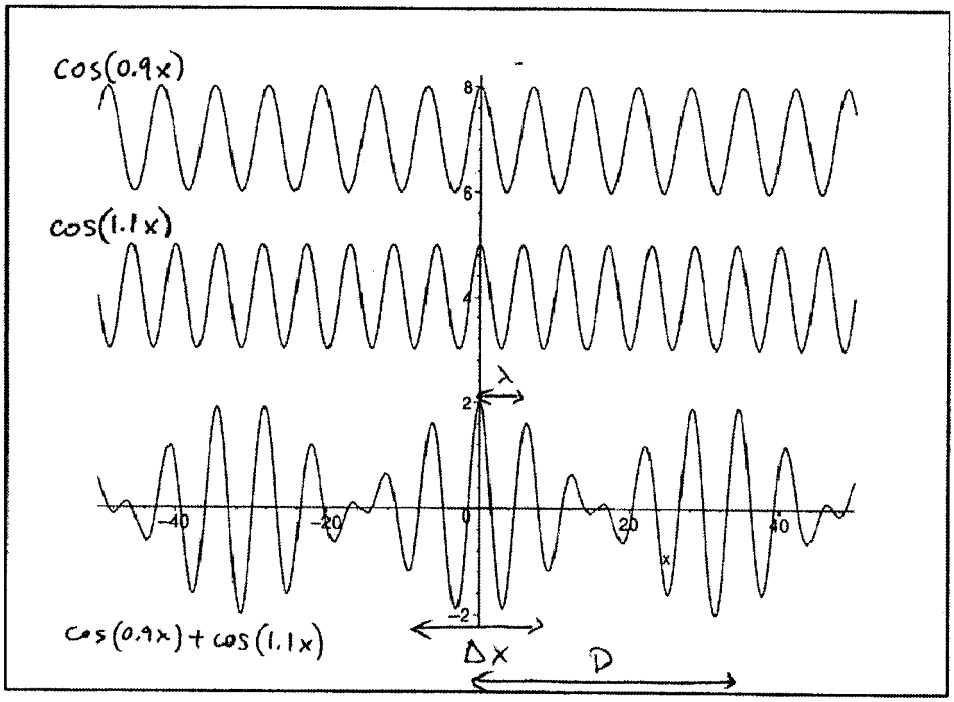


These traveling wave solutions to the Schrodinger equation do not represent real localized particles....let's see how we can do this. Let's look at Fourier series/transforms – these are **incredibly useful tools** in physics and engineering.

So we might expect that $\psi(x)$ for a traveling electron looks like a wavepacket:



Wavepackets can be built by adding up pure waves with a range of wavelengths centered around the wavelength λ. Let's take a closer look at how this works. We can start by adding up two different waves with wavelengths near a central value of $\lambda=2\pi$ (in some units).



We can see there are a series of wavepackets.

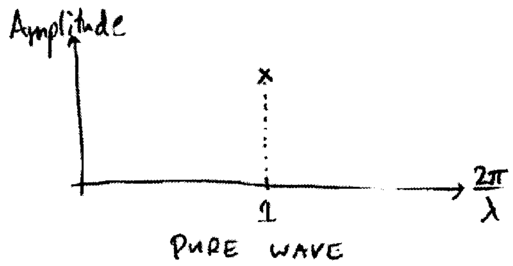
a) From the picture, estimate the wavelength, the size of each packet, and the separation between the packets:

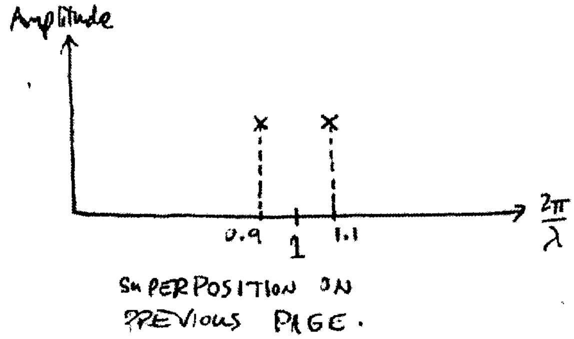
$\lambda=$ 6

$\Delta x=$ 20

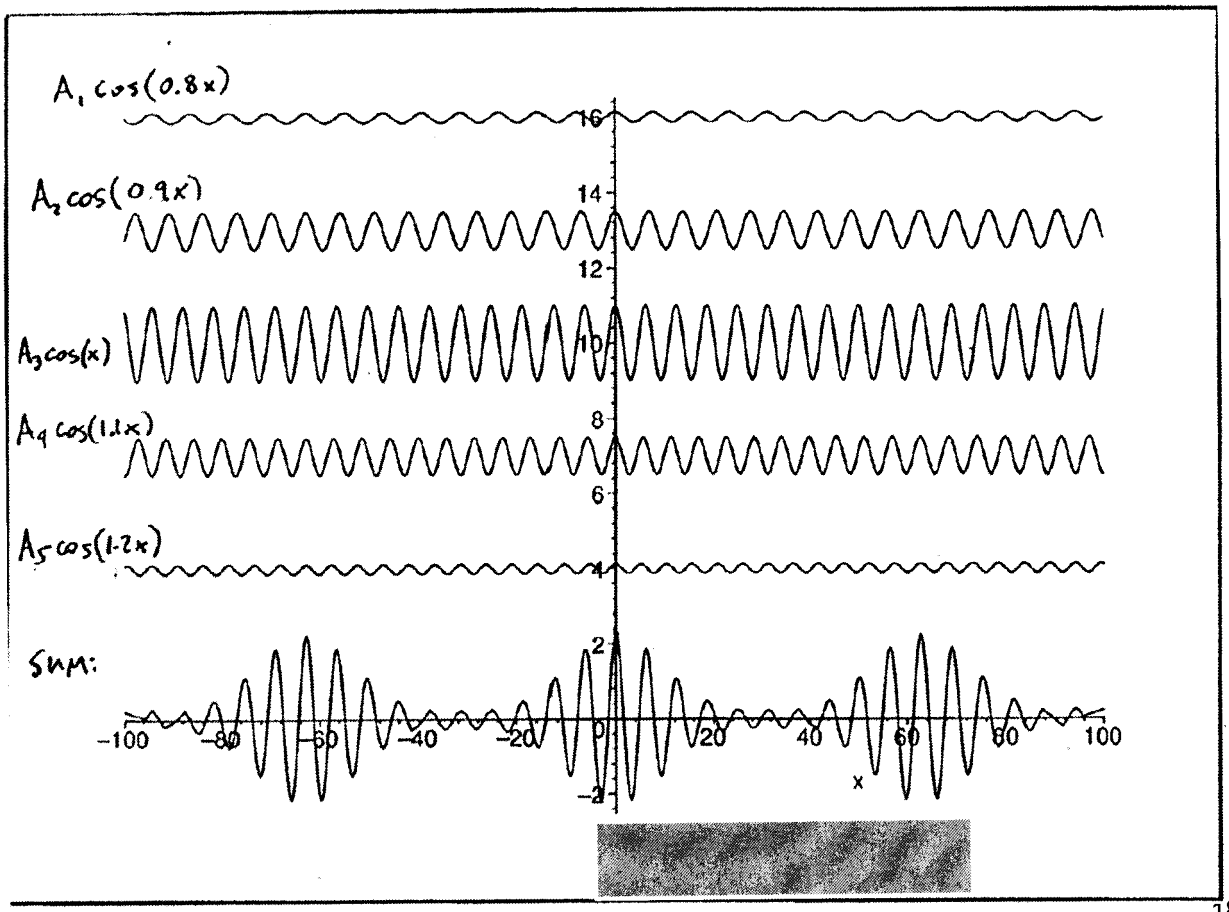
D= 30

We can represent the set of waves we are adding up by a diagram with the amplitude and the wavelength of each wave:





Now let's see what happens if we add more wavelengths:



Now for this superposition of **5 traveling waves**, estimate:

$\lambda =$ 6

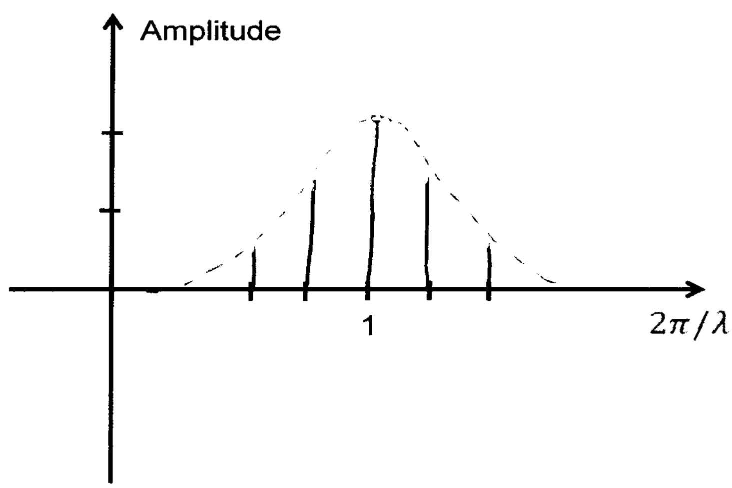
$\Delta x =$ 20 or 30

D= 65

What is the most significant difference from the previous case?

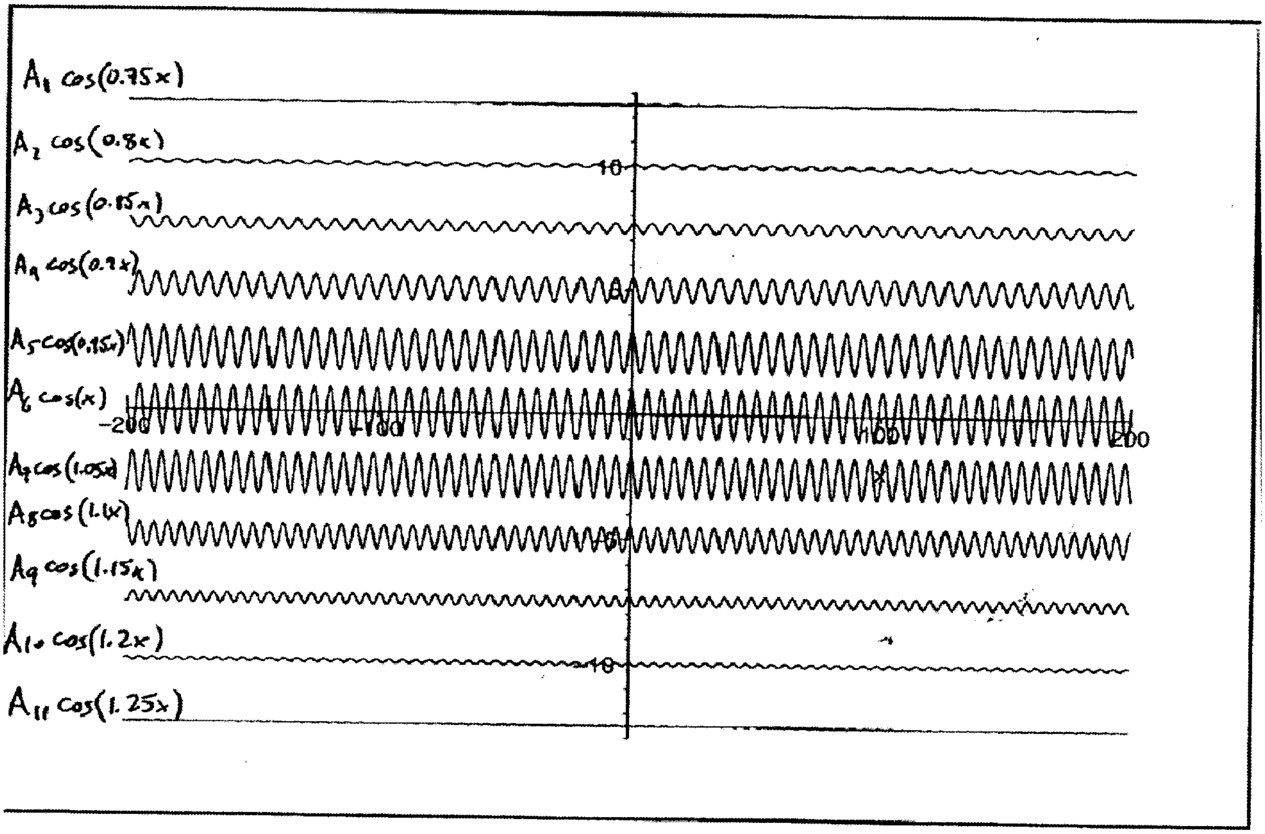
Wave packets are further apart.

Draw the amplitude/wavelength diagram for this superposition:

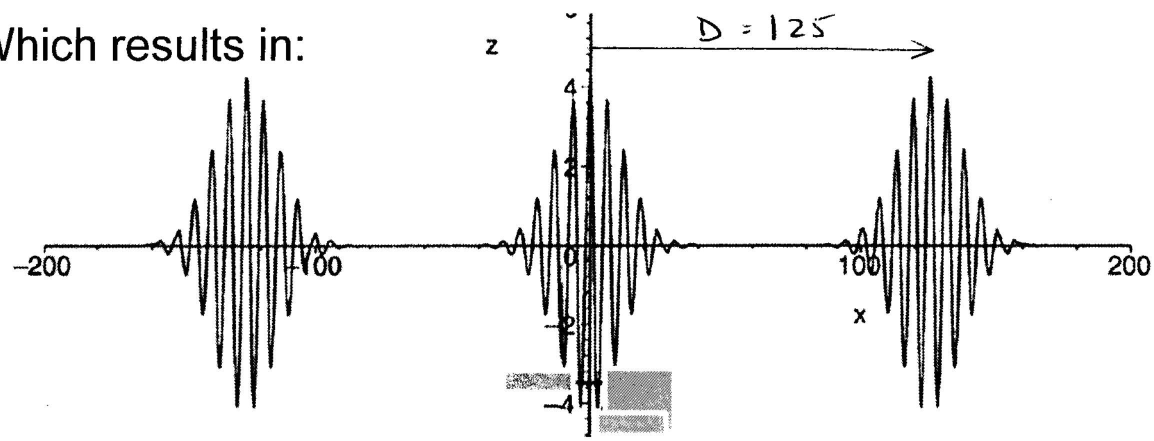


Notice that we have chosen the amplitude such that this diagram looks like a "bell curve" with a width (distance between points where the amplitude is ~ half of central value) of about 0.2.

Let's see what happens if we add even more waves, keeping the same width for our Bell curve:



Which results in:



Now for this superposition of **11 traveling waves**, estimate:

$\lambda = 6$

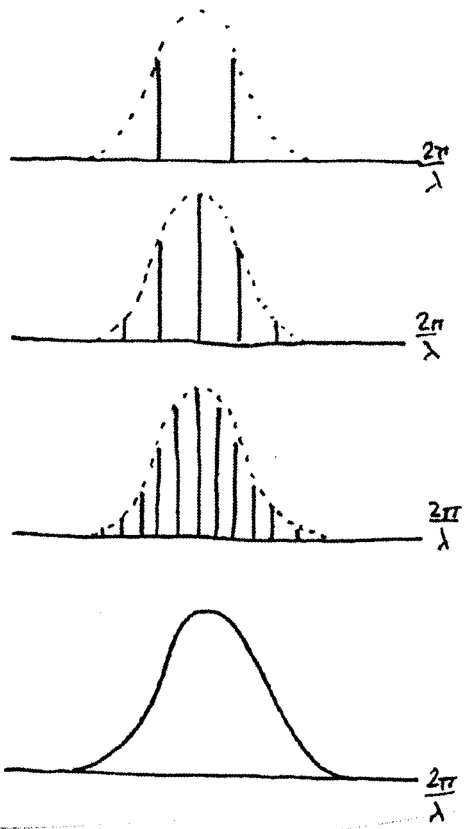
$\Delta x \approx 30$

$D = 125$

D=

What is the effect of adding more and more waves, keeping the same range of wavelengths $\Delta\lambda$ for our Bell curve?

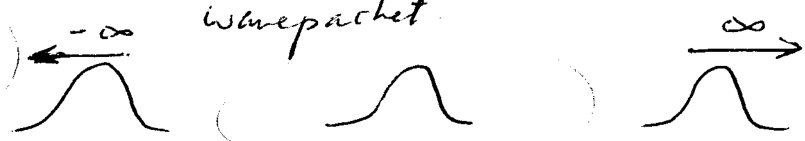
Wave packets move even further apart.



Based on the pattern you have observed, What do you think we would get if we added up an infinite number of wavelengths (using an integral) with the amplitude for a given wavelength determined by the same bell curve distribution that we have been using?

Answer:

Get a single isolated wavepacket



left with this pulse only.

Alright, let's look at the **Fourier: Making Waves simulation:**

From "Fourier: Making Waves" simulation

(100 points for questions 1-3)

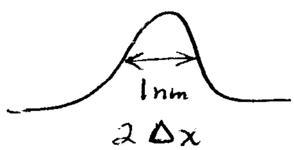
1- How are σ_x and σ_k related? When one quantity increases, what happens to the other one?

For short pulses (small σ_x), σ_k gets bigger and vice versa.

2- Give an estimate for the product $\sigma_x\sigma_k$.

$\sigma_x \sigma_k \sim 1$

3- Let's say we have an electron wavepacket with initial size σ_x =1nm, give an estimate for its size 1sec later. (Make sure to ask for help from the TA's)

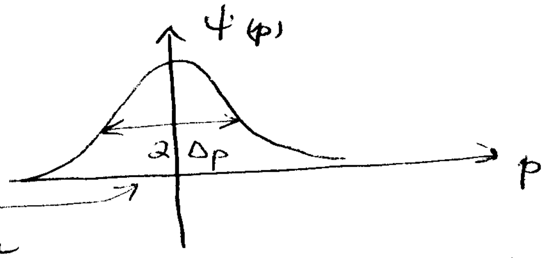


From $\Delta x \Delta k \sim 1$

We have $\hbar \Delta k = \Delta p$, where $\hbar \equiv \frac{h}{2\pi}$

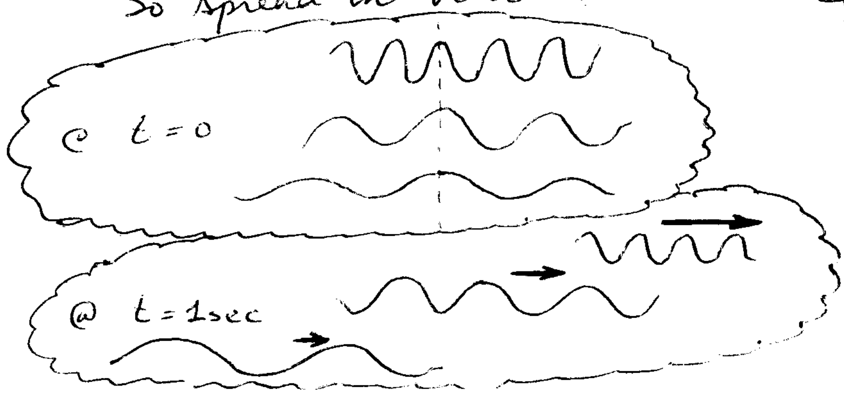
So spread in momenta $\Delta p \sim \frac{\hbar}{\Delta x}$

$$\Delta p \sim \frac{1.05 \cdot 10^{-34}\ J \cdot s}{10^{-9}\ m} = 1.05 \cdot 10^{-25} \sim 10^{-25}$$



For an electron at rest and $\Delta x = 1nm$, we need a significant spread in De Broglie wavelengths to synthesize such a short wavepacket.

So spread in velocities $\Delta v = \frac{\Delta p}{m} \sim \frac{10^{-25}}{9.11 \times 10^{-31}} \sim 10^5\ \frac{m}{s}$



$\Delta x(@ t \sim 1sec) \sim \Delta v\, t \sim \underline{10^5 m}$

Ans: $\sim 10^5 m$!