# MATH 152 - Learning Goals

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## 1 Course-level Learning Goals

After completing this course, students should be able to

1. recognize linear algebra questions (for which there are straight-forward analytic and numerical solution techniques) as parts of applied problems

2. make the connection between geometric properties and analytic quantities (determinants, dot and cross products, eigenvalues, etc.)

3. recognize that linear systems of equations can have unique, infinite or no solutions and know how to determine all solutions or that none exist

4. recognize matrix multiplication as a linear transformation and that such transformations (to the same dimensional space) can be simplified using eigen-analysis

5. use complex numbers, which arise naturally in the eigen-analysis of matrices

After completing the computer labs, students will

1. be familiar with a computational tool (MATLAB) that is commonly used in later courses and scientific careers

2. be able to solve larger, more interesting applied problems that would otherwise be inaccessible using analytic methods

# 2 Topic-level Learning Goals

## 2.1 Vectors and Geometry

Topics:

1. vectors: coordinate representation, length, dot product
2. projection
3. 2x2 and 3x3 determinants
4. cross product
5. lines and planes in 2D and 3D
6. application to computer graphics
7. geometry of solutions of linear systems
8. linear dependence and independence

Learning Goals:

- represent quantities that have a magnitude and a direction as vectors
- given a choice of coordinate axes, represent a vector as a column or a row using vector components
- represent the standard basis vectors in 2D and 3D both graphically and as column vectors
- perform vector addition and scalar multiplication
- compute the length of a vector given its components
- define unit vectors
- know the definition and basic properties of the dot product of two vectors
- compute the dot product of two vectors
- compute the angle between two vectors

- determine when two vectors are orthogonal
- find the projection of a given vector onto another vector
- express a 2D or 3D vector using the standard basis vectors
- know what a matrix is
- compute determinants in two and three dimensions
- explain the relationship between the rows of a 2x2 determinant and the value of the determinant
- know the definition and basic properties of the cross product of two vectors
- know that the length of the cross product axb is the area of the parallelogram spanned by a and b
- apply the cross product to rotational motion
- given three vectors, compute their triple product (a 3x3 determinant calculation).
- show the relationship between the triple product of three vectors and the volume of the parallelepiped spanned by the vectors
- find the parametric form and the equation form of a line in two and three dimensions
- find the parametric form and the equation form of a plane in three dimensions
- find whether a point is on a line/plane
- find the point of intersection of lines and planes
- find the line of intersection of planes
- describe the parametric and equation form of a point in two and three dimensions
- use either the parametric or the equation descriptions of lines and planes to solve simple geometrical problems

- provide a geometrical interpretation of a system of linear equations and its set of solutions in 2D and 3D
- know how to determine the view plane coordinates of position vectors in computer graphics applications
- know the definition of a linear combination of vectors
- give a parametric description of two and three dimensional space
- know the definition of linearly dependent and independent vectors
- give a geometrical interpretation of linear dependence in two and three dimensions
- know the definition of a basis

Computer Lab Learning Goals:

- enter vectors and matrices
- perform component-wise operations on matrices, using MATLAB operators `+, .*` etc.
- use MATLAB scalar functions `sqrt, sin, cos, tan, asin, acos, atan, atan2` etc.
- use MATLAB vector functions `dot, cross, sum, norm` etc.
- use MATLAB matrix function `zeros`

## 2.2 Systems of Linear Equations and Gaussian Elimination

Topics:

1. solving linear systems
2. elementary row operations
3. Gaussian elimination
4. homogeneous equations and the structure of solutions

5. geometric applications

6. application to resistor networks

Learning Goals:

- given a system of linear equations in n unknowns, perform elementary row operations (Gaussian elimination) to transform the system into an easily solved system and find the set of solutions
- know what the augmented matrix of a system of linear equations is
- recognize when a matrix is in row echelon form or reduced row echelon form and what operations are needed for augmented matrices in these forms to determine the solutions.
- determine when a linear system has a solution; when appropriate, write the set of solutions in parametric form and give a geometrical interpretation of the set
- know what the rank of a matrix is
- discuss the set of solutions of a linear system based on the rank of the corresponding augmented matrix
- discuss the set of solutions of an homogenous system of equations based on the number of unknowns and non-zero equations in the system
- establish a relation between solutions to homogeneous and inhomogeneous systems
- determine when a set of vectors is linearly dependent
- find the point or line of intersection of planes by solving a system of linear equations
- write down a linear system describing the relations between currents and/or voltages in a resistor network, determine the set of solutions to the system, and interpret the solution physically

Computer Lab Learning Goals:

- extract a sub-matrix or a particular entry in a vector

- find the reduced row echelon form of an arbitrary matrix using the MATLAB command `rref`
- interpret the solution or solutions (if any) to a linear system by reading the result of `rref` on an augmented matrix
- use MATLAB commands to find the solution to a system describing Kirchhoff's laws for a resistor network
- compute the voltage to current map matrix

## 2.3 Matrices and Determinants

Topics:

1. matrix multiplication
2. linear transformations
3. rotations, projections and reflections in 2D
4. matrix representation and composition of linear transformations
5. application to random walks
6. matrix transpose
7. matrix inverse
8. determinants

Learning Goals:

- perform matrix multiplication
- recognize matrix time vector multiplication as a linear combination of matrix column vectors
- be able to write a system of linear equations in matrix form
- know what a transformation (mapping) is
- show when a transformation is linear; use linearity to find transformed vectors

- analyze rotation, projection, and reflection in 2D as linear transformations of vectors
- interpret linear transformations as multiplication by a matrix
- find the matrix of a linear transformation given how the transformation act on the standard basis vectors
- compose linear transformations and express the composition in terms of matrix product
- use vectors and matrices to model random walks; use matrix multiplication to predict the probability that a system undergoing a random walk will be in a certain state at time n given the system's initial state
- know the definition and properties of the transpose of a matrix
- know the definition of inverse of a matrix and determine when a matrix is invertible
- compute the inverse of a matrix
- compute the determinant of an nxn matrix
- compute the determinant of a triangular matrix
- know the effects of row operations in computing determinants
- know about the linearity of determinants
- know the relation between matrix invertibility and determinants
- know product formula for determinants
- know how to compute the determinant of the transpose
- Cramer's rule for 2x2 systems

Computer Lab Learning Goals:

- use the MATLAB operator $*$ to multiply matrices
- use MATLAB to analyze the behaviour of random walks that are large or where the interest is at long times

- use the MATLAB operator `\` to find a solution to a linear system
- use MATLAB function `eye` (identity matrix)
- compute the inverse of a matrix A using MATLAB command inv(A)
- know the MATLAB commands `'` for transpose and `det` for determinants
- create, edit, and run m-files
- write scripts involving `for` loops

## 2.4 Eigenvalues and Eigen-vectors

Topics:

1. eigenvalues and eigen-vectors
2. complex numbers
3. complex eigenvalues and eigen-vectors
4. diagonalization, powers of a matrix;
5. application to random walks;
6. complex exponential
7. systems of linear differential equations
8. vector differential equations

Learning Goals:

- Know the definition of eigenvalues and eigen-vectors of a matrix and their geometrical interpretation
- Compute eigenvalues by calculating the zeros of the characteristic polynomial and find eigen-vectors by determining the non-zero solutions of the corresponding system.
- find a basis of eigen-vectors when it exists

- interpret complex numbers as points on a plane and identify real and imaginary part
- compute the modulus and the complex conjugate of a complex number
- perform basic algebra with complex numbers: addition, multiplication, fraction simplification
- find complex eigenvalues and eigen-vectors when the characteristic polynomial has complex roots
- Know the relationship between the complex exponential and the unit circle in the complex plane
- add and multiply complex exponentials
- establish relationships between the eigenvalues of a diagonal matrix, the matrix's entries and determinant
- use eigenvalues and eigen-vectors to perform matrix diagonalization
- use eigenvalues to compute high powers of a diagonalizable matrix and to calculate the determinant of a matrix
- use eigenvalues and eigen-vectors to explore the time evolution of a random walk
- convert a system of differential equations into a vector differential equation and find the general solution to the equation
- find the solution of an initial value problem for a system of differential equations

Computer Lab Learning Goals:

- use MATLAB command `eig` to find eigenvalues and eigen-vectors
- use MATLAB eigen-analysis to determine solutions of differential equations
- use the MATLAB `plot` command to plot solutions of vector differential equations