Math 307: homework problems 7

1. For the following matrices find
   (a) all eigenvalues
   (b) linearly independent eigenvectors for each eigenvalue
   (c) the algebraic and geometric multiplicity for each eigenvalue

   and state whether the matrix is diagonalizable.

$$A = \begin{bmatrix} 3 & 7 \\ 2 & -2 \end{bmatrix} \quad \textbf{(calculate by hand)}$$

$$B = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \quad \text{(calculate using Matlab/Octave or otherwise)}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & -2 \\ -1 & 2 & 3 \end{bmatrix} \quad \text{(calculate using Matlab/Octave or otherwise)}$$

2. Find a 3×3 real, non-zero (*i.e.* not all entries zero) matrix which has all three eigenvalues zero.

3. (a) **By hand** find a matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$ and corresponding eigenvectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

   (b) Using Matlab/Octave or otherwise, find a matrix with eigenvalues $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$ and corresponding eigenvectors

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 9 \\ 4 \\ 4 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}.$$

4. (Bonus) Show that if A is an $n \times n$ square matrix and each column sums to c, then c is an eigenvalue of A. *Hint: if you cannot show this in a few lines, try another approach.*

5. (Bonus) If $p(\lambda)$ is the characteristic polynomial of an $n \times n$ invertible matrix A, find an expression for the characteristic polynomial of A^{-1} in terms of the characteristic polynomial of A.

6. Find the Jordan canonical form of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. The matrix

$$A = \begin{bmatrix} 6 & 1 & 2 & 2 & 1 \\ 1 & 5 & 2 & 1 & 2 \\ 2 & 2 & 3 & 1 & 2 \\ 2 & 1 & 1 & 3 & 2 \\ 1 & 2 & 2 & 2 & 3 \end{bmatrix}$$

has positive eigenvalues. Use the power method to find the largest and the smallest ones, and the corresponding eigenvectors. Check whether the two eigenvectors you have computed are orthogonal.