

Math 307: homework problems 6

1. Compute the Fourier coefficients (c_n 's, a_n 's and b_n 's) for the triangle function

$$f(t) = \begin{cases} 2t & \text{if } 0 \leq t \leq 1/2 \\ 2 - 2t & \text{if } 1/2 \leq t \leq 1 \end{cases}$$

and show that the Fourier series decomposition of $f(t)$ may be written

$$f(t) = \frac{1}{2} - \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4}{\pi^2 n^2} \cos(2\pi n t) = \frac{1}{2} - \sum_{n=0}^{\infty} \frac{4}{\pi^2 (2n+1)^2} \cos(2\pi(2n+1)t)$$

What does Parseval's formula say in this case?

2. Modify the file `ftdemo1.m` or `ftdemo2.m` so that it plots the partial sums of the Fourier series in question 1. Hand in a plot (or a sketch) of the partial sums with 1, 2, 5 and 10 non-zero terms.
3. Compute the Fourier coefficients (c_n 's, a_n 's and b_n 's) for the half sine wave

$$f(t) = \sin(\pi t) \quad \text{for } 0 \leq t \leq 1$$

and show that the Fourier series for $f(t)$ can be written

$$f(t) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{1-4n^2} \cos(2\pi n t)$$

4. Compute the Fourier coefficients (c_n 's, a_n 's and b_n 's) for the function

$$f(t) = t^2 - 1 \quad \text{for } -1 \leq t \leq 1$$

and show that the Fourier series for $f(t)$ can be written

$$f(t) = -\frac{2}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(n\pi t).$$

5. (Bonus) Show that the Fourier series of $f(t) = e^t$ on the interval $-\pi \leq t \leq \pi$ is

$$f(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \frac{1}{1-in} (e^{(1-in)\pi} - e^{-(1-in)\pi}) e^{int}.$$

Deduce that

$$\sum_{n=1}^{\infty} \frac{1}{1+n^2} = \frac{1}{2}(\pi \coth \pi - 1).$$

6. Compute the discrete Fourier transform of $\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ using (i) the fast Fourier transform and (ii) the matrix product.

7. Compute the discrete Fourier transform of $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ using whichever method you would prefer.

8. The file `dutch_harbor_tides.m` contains the tidal height in millimetres at Dutch Harbor, Alaska. The data is at hourly intervals from 1985 to 2005. Find the frequency, amplitude plot for the tides (hand in a plot for the range 8°hr^{-1} to 32°hr^{-1} or a list of the MATLAB/Octave commands you used to find it). Does Dutch Harbor have a predominantly twice-daily or daily tide?