# Math 307: homework problems 2

Many problems in this homework make use of a few MATLAB/Octave `.m` files that are provided on the website. In order to use them, make sure that the files are in the same directory that you are running MATLAB/Octave from (to see which directory this is, type `pwd` in MATLAB/Octave).

1. Derive the matrix equation to solve in order to find the cubic spline passing through the three points $(0, 1)$, $(0.5, 2)$ and $(1, 4)$. Plot the resulting spline (you may use the file `plotspline.m`).

2. What happens to the condition number of the matrix $S$ used in cubic spline interpolation as the size $n$ becomes large (you may use the file `splinemat.m`)?

**Choose one of questions 3 and 4**

3. A parabolic runout spline is the interpolating function you get by changing the condition $f''(x_1) = f''(x_n) = 0$ to the condition that $p_1(x)$ and $p_{n-1}(x)$ should be quadratic polynomials (that is, $a_1 = a_n = 0$). Modify the file `splinemat.m` so that it computes the matrix relevant to this modified problem. Call the modified file `splinematpr.m`. (Hand in a description of your changes, or a print-out of the modified file.) Use your new file to graph the parabolic runout spline for the points $(1, 1)$, $(2, 1)$, $(3, 2)$, $(4, 4)$ and $(5, 3)$. (The easiest way to do this is to change `splinemat` to `splinematpr` inside the file `plotspline.m` and call the modified file `plotsplinepr.m`. Use this new file to plot the modified spline.) Hand in a plot of both the parabolic runout spline and the cubic spline on the same graph.

4. Consider the problem of interpolating four points $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$ and $(x_4, y_4)$ with a function $f(x)$ that is given by a quadratic polynomial in each interval $x_i, x_{i+1}$, (i.e., $p_i(x) = a_i(x-x_i)^2 + b_i(x-x_i) + c_i$) and whose first derivative $f'(x)$ is continuous across the points $x_i$. Write down the system of equations for this problem. Is there a unique solution to this problem?

5. Derive the matrix equation to solve in order to find the finite difference approximation with $n = 4$ for the differential equation
$$f''(x) + xf(x) = 0$$
subject to
$$f(1) = 1, \quad f(3) = -1.$$

The following questions consider the steady heat equation in a one-dimensional rod considered in lectures:

$$0 = kT''(x) - HT(x) + S(x),$$

where $k$ and $H$ are constants, subject to the boundary conditions

$$T = T_l \text{ at } x = x_l \text{ and } T = T_r \text{ at } x = x_r.$$

The MATLAB/Octave commands needed to find the finite difference approximation for $T(x)$ in the case $k = 1,\ H = 0,\ S(x) = 1,\ T_l = T_r = 1,\ x_l = 0$ and $x_r = 1$ are provided in `heat.m`.

6. Modify the commands provided in `heat.m` to calculate the temperature profile in a rod cooled by the air in the case $k = 1$, $H = 1$, $S(x) = 1$, $T_l = 0$, $T_r = 2$, $x_l = -1$ and $x_r = 1$. Hand in a plot of the solution for $n = 50$.

7. For the case given in Q6, compute the finite difference approximation at $x = -0.5$ for $n = 4$, 40 and 400. The true solution at this point is $1 - \sinh 0.5/\sinh 1$. Make a log-log plot of the magnitude of the error in the finite difference approximation against $\Delta x$. What is the approximate slope of this curve?

*Optional* The boundary condition $T'(x) = 0$ at $x = x_l$ or $x = x_r$ describes an insulating end to the rod. Write down an approximation for $T'(x_l)$ using $T_0$ and $T_1$. Also write down an approximation for $T'(x_r)$ using $T_{n-1}$ and $T_n$. Find the modification needed to the matrix equation if insulating boundary conditions are placed at $x = x_l$ and $x = x_r$ (you should find that two rows of the matrix change and two entries of the vector on the right-hand-side change). Modify the commands provided in `heat.m` to calculate the temperature profile in a heated rod in the case $k = 1$, $H = 0$, $S(x) = 1$, with insulating boundary conditions at $x = 0$ and $x = 1$ (representing a continuously heated rod from which no heat escapes). Try to find the solution for $n = 10$. Is the solution reasonable?