Ex Suppose a cylindrical water tank of height 1 m with cross-sectional area $1\,m^2$ is being filled at a rate of 1 l/s with water. Water leaks out a small hole in the bottom. When the tank is completely full, the leak rate is 2 l/s. Assume the draining tank obey's Torricelli's Law.

a) Can the tank ever empty?

b) Can the tank ever overflow?

c) Is there a stable, equilibrium water depth?

a) Write the DE for $h(t)$, the water depth.

Ex Derive the general equation for $h(t)$ in terms of the tank area A, maximum depth D, inflow rate Q_i, outflow rate at maximum depth Q_o. Scale the resulting equation to be as simple as possible.

5 Minute Mathematics Break.

A computational method is used to determine a drag coefficient for flow over a proposed aircraft wing design. Computations are first done on a design for which the drag is known exactly. The following errors are seen as the grid spacing h is refined:

h	Error
0.1	0.04002
0.05	0.00881
0.025	0.00201
0.0125	0.00049

What is the order of convergence of the method?

Ex Approximate the solution of

$$y' = \sqrt{y^2 + t^2}$$

with $y(0) = 1$ at $T = 0.2$ using 2 steps of FE with step size $h = 0.1$.

Ex Consider the DE system

$$\frac{dx}{dt} = x + y + t$$

$$\frac{dy}{dt} = x - y.$$

with initial data $x(0) = 1$, $y(0) = 0$.
Use Euler's method with 2 steps of size $h = 0.1$ to approximate the solution at $t = 0.2$ (both $x(0.2)$ and $y(0.2)$)

5 Minute Mathematics Break.

Consider the following DE for $x(t)$:

$$\dot{x} = ax - bx^2 + cx^3. \qquad (a, b, c \text{ given positive}).$$

Scale x and t to make the first two terms on the RHS of the equation as simple as possible.

You should have a single parameter left in the equation, a combination of a, b, and c.

5 Minute Mathematics Break.

Convert the following 2nd order DE for $x(t)$ to a first order system:

$$\ddot{x} + t^2\dot{x} + x^2 = 0.$$

If you have time, apply FE stepping to this problem with

$$x(0) = 1, \quad \dot{x}(0) = 1$$

Do 2 steps of size $h = 0.1$.

Ex Consider the problem for $y(t)$:

$$\frac{dy}{dt} = t + y^2 \quad \text{with } y(0) = 1.$$

Approximate $y(0.1)$ using one step of BE time stepping.

Ex Use one step of Improved Euler to approximate $y(0.2)$ where $y(t)$ solves

$$y' = 3 - t + y$$

with $y(0) = 1$.