Tutorials week 8 - Math for Mech 221
Brian Wetton (wetton @ math. ubc. ca).

This week we're covering first order,
linear, homogeneous systems.

$$\underline{x}' = A\underline{x}.$$

Note: we only consider the case where
A is diagonalizable, no repeated roots
with rank-deficient eigen spaces.

A) Solve a bunch of 2×2 problems
$$\underline{x}' = A\underline{x}, \qquad \underline{x}(0) \text{ given.}$$

have A with 2 real, distinct eigenvalues
and A complex conjugate   "
identify the behaviour at $\underline{x} = \underline{0}$
(stable, unstable, centre).

B) As time permits, solve some 3×3
problems, where one eigen value is
given or can be guessed.

Some examples:

a) Find the general solution of

$$x_1'(t) = 2x_1 + x_2$$

$$x_2' = x_1 + 2x_2.$$

Then, Find the solution of this problem when

$$x_1(0) = 2, \qquad x_2(0) = 1.$$

b) Solve $\quad x_1' = x_2$

$$x_2' = -x_1.$$

$$x_1(0) = 2, \qquad x_2(0) = -2.$$

Note: This is the undamped spring $x_1'' + x_1 = 0$ problem written as a system.

c) $\quad y' = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} y \qquad y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$