Mech 2 Math Suggested Problems Week #1

Brian Wetton

September 15, 2008

Note: Questions #2 and #3 might be a bit tricky. Feel free to ask me or the TA for help with them.

1. Experimental Measurements determine that a function $f(x)$ satisfies $f(0) = 0$, $f'(0) = 1$ and $f(1) = 2$.

   (a) Estimate $f(1/3)$ using tangent line (linear) approximation.

   (b) Estimate $f(1/3)$ using linear interpolation.

   (c) Estimate $f(1/3)$ more accurately using all three pieces of information given. **Hint:** Construct a quadratic polynomial $Q(x)$ that satisfies the data. Approximate $f(1/3)$ by $Q(1/3)$.

2. A numerical method is used to approximate an improper integral. The method converges as the number of subintervals N gets larger, but convergence is slow. The estimates, E_N of I for various values of N are given below:

N	E_N
4	1.5250
8	1.3661
16	1.2563
32	1.1799

   (a) Estimate the order of convergence. This is the number r such that the error is approximately C/N^r for some constant C when N is large. **Hint:** If

   $$I - E_N \approx C/N^r$$

   then

   $$E_{2N} - E_N \approx C(1 - 2^{-r})/N^r$$

and so

$$\frac{E_{2N} - E_N}{E_{4N} - E_{2N}} \approx 2^r$$

or

$$r \approx \log_2 \left(\frac{E_{2N} - E_N}{E_{4N} - E_{2N}} \right).$$

Use this last expression to estimate r.

(b) Use an appropriate Richardson Extrapolation to get more accurate estimates of I.

3. Suppose that linear interpolation $S(x)$ is used to estimate $f(x)$ in the interval $[0, h]$ using the values $f(0)$ and $f(h)$. Derive an error bound for this interpolation of the form

$$|f(x) - S(x)| \leq CK_2h^2$$

for all x in $[0, h]$ where K_2 is the maximum of $f^{(2)}$ on the interval and C is a constant you must determine. **Hint:** modify the argument I showed in class for the error bound for tangent line approximation.