Mech 221 Math lecture 2 Notes

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I. Rollé's Theorem: If $f(x)$ is differentiable and $f(a)=0$ and $f(b)=0$ then there is a point ξ in (a,b) at which $f'(\xi)=0$.

Proof: there are three cases:

(i) f has some positive values in $[a,b]$.

(ii) f has " negative " " "

(iii) f is identically zero in $[a,b]$.

Case (i) The maximum of f on $[a,b]$ must then be positive and so can't be attained at the end points. Therefore, at a point ξ where the maximum is attained, $f'(\xi)=0$.

Case (ii) similar argument on minimum.

Case (iii) $f'(x)=0$ at all points x in $[a,b]$.

II. Use Rollé's theorem to prove the remainder expression for linear approximation (Taylor approximation of order $n=1$).

Pick the base point a and a particular value of x - call it b - where the approximation is made.

We want to show that

$$f(b) - [f(a) + f'(a)(b-a)] = \frac{f''(\xi)}{2}(b-a)^2 \quad ✪$$

for some ξ in $[a,b]$. Consider the somewhat tricky function

$$g(x) = f(x) - [f(a) + f'(a)(x-a)] - (x-a)^2 \underbrace{\frac{1}{(b-a)^2}\{f(b) - [f(a) + f'(a)(b-a)]\}}_{\text{does not depend on } x, \text{ a big constant } K.}$$

$g(a) = 0$ and $g(b) = 0$ so (Rollé)
$g'(\xi_1) = 0$ for some ξ_1 in $[a,b]$.

$g'(x) = f'(x) - f'(a) - 2(x-a)K.$

So $g'(a) = 0.$

Rollé again, $g''(\xi) = 0$ for ξ in $[a, \xi_1]$
(we can say ξ in $[a,b]$).

$g''(x) = f''(x) - 2K.$

writing out $g''(\xi) = 0$ gives ✪.