Mech 221: Computer Lab 4

Hand in the solutions to the three questions in the lab at the *end* of the lab.

Question 1: Using `ode45`

You will work through an example which should illustrate the use of `ode45`, working through several steps to ensure that your code is computing the right solution. Recall from lab # 2, you implemented the Forward Euler method to solve scalar differential equations. In this question, you will modify your Forward Euler code to solve a vector equation as one of the steps to check your code.

Recall from the pre-lab that we found the system that corresponds to the following IVP:

$$\ddot{y} + y = 0, \quad y(0) = 1, \dot{y}(0) = 0 \tag{1}$$

By defining $y = x_1, \dot{y} = x_2$, we found that we could transform this problem into the following

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} x_2 \\ -x_1 \end{pmatrix}, \quad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

where we have written the equation and initial conditions as column vectors as used by MATLAB. The solution to the original problem is $y(t) = \cos t$ so the solution of the system is $x_1(t) = \cos t$ and $x_2(t) = \sin t$ (recall that $x_2 = \dot{y}$).

- Create an .m file function "`vfunc.m`" that will return the value of the right hand side of the DE system. Make sure the first line reads as follows:

  ```
  function f = vfunc(t,x)
  ```

  The reason for this is that `ode45` requires the function being passed to it to accept input in the order (independent variable t, dependent variable vector $\vec{x}$ in a column). Your `vfunc.m` should return a *column* vector of length 2. When your code is working below, *print out your* `vfunc.m` *file.*

- Now modify your Forward Euler code so that it can solve a system with two components, calling `vfunc.m` to give the RHS vector.

- Using your modified Forward Euler code, find the numerical solution to the system above over the interval $0 \le t \le 3\pi$, with the given initial condition using $N = 100$ sub-intervals ($h = 3\pi/100$). Plot both components of your numerical solution and the exact solution on the same graph with a `legend` as described in the pre-lab. *Print out this graph.* You should get reasonable (not great) agreement between the numerical and exact solutions. This gives confidence that your function `vfunc.m` is implemented correctly.

- Now you will solve the same system using `ode45` over the interval $0 \le t \le 3\pi$, with the given initial condition. Assign a variable `x0` to be the initial condition for the system, as given above. Remember, `x0` must be a column vector as discussed in the pre-lab. Now type in the following command:

  ```
  >> [t,x] = ode45('vfunc',[0 3*pi], x0);
  ```

  Calling this will return two variables `t,x`. Notice that `x` is a matrix. How the two components of the solution $\vec{x}$ are stored in this matrix is discussed in detail in the pre-lab. Plot both components of your numerical solution and the exact solution on the same graph with a `legend`. *Print out this graph.* This numerical solution should be *very* accurate.

- *Hand in the print-out of* `vfunc.m`*, the plot of your FE approximation compared to the exact solution, and your plot of the* `ode45` *approximation compared to the exact solution.*

By getting agreement to a known solution, you should have confidence that you are calling `ode45` correctly. You will need to do similar calculations for the remainder of the lab, so keep your `vfunc.m` file handy. You will wish to modify it for future calculations.

Question 2: Using `ode45` on the RLC circuit equations

Recall that we considered in the pre-lab the system of equations describing a RLC circuit:

$$\begin{aligned} \dot{V} &= \tfrac{1}{C} I \\ \dot{I} &= -\tfrac{1}{L} V - \tfrac{R}{L} I. \end{aligned} \qquad (3)$$

with initial conditions $V(0) = 2$, $I(0) = 1$.

- With $(R, L, C) = (4, 2, 0.5)$, modify `vfunc.m` so that you could use it to solve this pair of differential equations using `ode45`.

- Now use `ode45` to solve this problem. Recall the analytical solution was $I(t) = e^{-t} - 2te^{-t}$ and $V(t) = 2e^{-t} + 4te^{-t}$. Plot both your numerical solution for $V(t)$ and $I(t)$ and the exact solution on the same graph with a `legend`. *Print out this graph.* Again, the numerical solution should be very accurate.

- *Hand in your plot of the* `ode45` *approximation compared to the exact solution.*

Question 3: A Third Order Mystery Equation

When you get to your lab, your group will be given a card on which is written a third order differential equation for $y(t)$ and initial conditions $y(0)$, $\dot{y}(0)$ and $\ddot{y}(0)$. You will not be able to find the exact solution of the DE. You will solve this problem numerically and hand in a plot of your approximation of $y(t)$ in the interval $0 \leq t \leq 1$.

- Convert the third order equation to a first order system with three components.

- Modify `vfunc.m` to return the three components of the RHS of your system above given inputs of t and the three components of the solution.

- Solve the problem approximately using `ode45`. Plot your approximation of $y(t)$ over the interval $0 \leq t \leq 1$. It is not necessary to plot all components of the solution (the other components will be derivatives of y). *Print out your plot.*

- *Hand in the plot of your approximate solution. Attach your DE problem card to your plot. Your mark for this question is entirely dependent on having the correct (approximate) solution to the given problem.*